A SIMULATION MODEL OF BALLAST SUPPORT
AND THE MODULUS OF TRACK ELASTICITY

by
J. R. Lundgren
G. C. Martin
and
W. W. Hay

conducted by
RAILWAY RESEARCH
DEPARTMENT OF CIVIL ENGINEERING
ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS
in cooperation with
THE PENN CENTRAL TRANSPORTATION COMPANY

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This report was prepared as a master's thesis by Mr. J. R. Lundgren, under the direction of Dr. W. W. Hay.

The project has been sponsored by the Penn Central Transportation Company and the University of Illinois Research Board.
This report presents a theoretical method for the analysis of the behavior of the track structure under static load from which an evaluation of the track modulus can be made. The method simulates the track structure as a finite element model. The behavior of cohesionless soil materials under tensile and shear loading is included in the simulation. A computer solution of the model system by the methods of matrix structural analysis is given. Wheel loads, material constants and boundary conditions are applied to the model and a solution for deflections, stresses and track modulus is obtained. The method is illustrated by an example problem and the results of several determinations of track behavior under load are presented.
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1. INTRODUCTION

1.1 General

Modern, high capacity railroad operations require a solid foundation. The advent of higher speeds, greater traffic densities, and heavier wheel loads is adding a new dimension to the economic and engineering problems of track maintenance. One solution, justified by past practice, but with questionable support in theory, is the recourse to heavier rail. This is an expensive solution. The alternative of improving the stability of rail support has received too little attention in many quarters. Very heavy traffic conditions will require a careful combination of both solutions.

A method for investigating the behavior of the rail under load was successfully developed by Professor A. N. Talbot and is well documented in the Proceedings of the American Railway Engineering Association. The influence of rail moment of inertia and modulus of elasticity are related to rail moments, deflections and shears by the equations derived from the analysis of the rail as a continuous beam resting on a continuous elastic support. The remainder of the track structure is represented by a foundation spring constant, the track modulus, in this approach.

Little attention has been given to modeling the entire track structure. An analysis relating ties, tie spacing, wheel loadings and the material constants of ballast and subgrade to the stresses and deflections of the system would be of value. The investment in the track structure and the cost of its maintenance would attach considerable importance to such a model. The development of such a model is the purpose of this investigation.
1.2 The Modulus of Track Elasticity

Stability of track support is closely related to and is usually measured by the modulus of track elasticity, or as it is more commonly known, track modulus. Many factors will influence the value of track modulus: among the most significant are tie type and dimensions; ballast type, depth and stiffness; and subgrade type and stiffness. Track modulus is defined as the load per unit length of rail required to depress the rail one unit. Track modulus is usually expressed in terms of pounds per inch of load on the rail per inch of rail deflection (lbs/in/in). A basic formula for the evaluation of track modulus is:

\[ u = \frac{P}{Y} \]

where:

- \( u \) = value of track modulus
- \( P \) = uniform load per unit length of rail
- \( Y \) = track depression.

Values for the track modulus are often obtained by solving the equation for track deflections that has been derived from the analysis of a continuous beam resting on a continuous elastic support based on the work of Talbot:

\[ u = \frac{3}{\sqrt[3]{\frac{P^4}{64 E I Y_0^4}}} \]

where:

- \( u \) = modulus of track elasticity
- \( P \) = magnitude of single wheel load
- \( Y_0 \) = rail deflection under load point
- \( E \) = modulus of elasticity of rail steel
- \( I \) = moment of inertia of the rail section.
A more detailed discussion of the computation of track modulus with illustrative diagrams are presented in Section 3.3 of this report. It is to be realized that the prime purpose of Talbot's work was to investigate rail stresses without a detailed investigation of the theoretical aspects of track modulus.

The values of deflections used to solve for the track modulus are generally obtained from field tests. Field tests are not easy to perform and present particular difficulties when a wide range of track conditions or constructions are to be compared. The effects of different types and degrees of support are thus difficult to evaluate, especially in advance for design purposes.

1.3 Physical Characteristics of the Track Structure

The track structure consists of four basic parts: rail, ties, ballast and roadbed. This combination acts as an elastic structural system upon the application of wheel loads. The rail acts as a continuous beam on elastic supports, distributing load to a number of ties. The ties support the rail and distribute the load to the ballast along their length by beam action. The ballast transmits and distributes the load to the roadbed, the foundation of the structure.

The track structure responds to the applied wheel loads. The relationship between the load system and track behavior is an inseparable one; changes in one will affect the other.
1.4 Problem Approach

An analytical approach to the determination of the track modulus is of considerable interest and value. By relating the factors which enter into the consideration of track modulus in a mathematical way, a model which simulates the track structure may be created. Analyses of track under static load may then be performed and the appropriate value of track modulus under a given set of conditions determined. The components of rail support; subgrade, ballast section, and ties, comprise a layered sequence, each with its own modulus of stiffness and each encompassing a range of non-uniformity. These non-uniformities, a practical feature of the components of the system, may be accounted for with an appropriate statistical analysis.

The development and use of a suitable mathematical model which accounts for and includes the particular behavior of the individual components in the analysis of the gross behavior of the complete system to determine track support is the primary concern of this paper.

2. OBJECT AND SCOPE

The object of this investigation is to analyze the factors entering into the track modulus and to develop a systematic numerical procedure relating these parameters. This relationship will permit the simulation of track under load and will enable determination of the appropriate value for the track modulus under a given set of conditions. An attempt to develop a model system which will incorporate the various track structure components in an evaluation of the stresses and deflections under static rail loading has been made.

The complex nature of the track structure components render a closed form solution a practical impossibility. It is necessary to greatly simplify the actual nature of the problem in order to arrive at an analysis which yields a mathematically tractable solution.

In this study a two-dimensional finite element approximation of the track structure has been developed; a three-dimensional simulation proved unwieldy at this time. The plane model is used to represent a longitudinal section of the track structure. The model has the capability of accepting random assignments of soil properties established by the user. With this feature, some approximation to the variability found in natural soils may be included in the analysis. It is assumed that the user has a knowledge of the material properties of the system he wishes to analyze.

The analysis developed utilizes a finite element method of approach which transforms the problem of a continuum into one involving digital simulation by computer.
3. IMPORTANCE OF TRACK MODULUS

3.1 General

The value of the track modulus has been of key importance in track design problems. The importance of track modulus as a parameter of track performance is evident in the Talbot solution for stresses in the track structure, specifically in the rail. In his analysis, Talbot computes stresses in the rail on the assumption that the rail is a continuous beam supported on a continuous elastic foundation. The basis of the rail stress calculation is presented below:

Rail moment equation;

\[ M = P \sqrt[4]{\frac{E I}{64 u}} \]

Bending stress equation;

\[ f = \frac{M c}{I} \]

Where;

\[ M_0 = \text{maximum moment in the rail} \]
\[ E = \text{modulus of elasticity of rail steel} \]
\[ I = \text{moment of inertia of the rail section} \]
\[ P = \text{magnitude of single wheel load} \]
\[ u = \text{modulus of track elasticity} \]
\[ f_0 = \text{maximum fibre stress in bending} \]
\[ c = \text{distance from neutral axis to outermost fibre} \]

Further studies into stresses in the track structure have used a similar approach. Timoshenko and Langer\(^1\) utilized the beam on elastic foundation theory to develop an experimental method for the determination of vertical and lateral forces produced in rails by moving locomotives. Criner and McCann\(^2\) developed an elastic-analog computer technique for the
analysis of beams on elastic foundations that are subjected to high speed traveling loads. Each of these analyses is dependent upon either experimentally determined or estimated values of the track modulus.

Typical values of track modulus are shown in Table 1, demonstrating the practical importance of a knowledge of the track modulus.

3.2 Definition of Track Modulus

Three names are in use for the designation of track stiffness; track modulus, modulus of track elasticity, and modulus of elasticity of rail support. Although the others are more descriptive, the term track modulus will be used in this report to designate the numerical value representing track stiffness.

Track modulus is a measure of the stiffness of support offered to a rail under load. Track modulus, denoted by the symbol \( u \), is the ratio of applied load to track depression; the load per unit length of each running rail required to depress the track one unit. This relationship is shown schematically in Figure 1. The definition prescribed by Talbot in his work with the Special Committee on Stress in Railroad Track is as follows:

\[
\text{"u = an elastic constant which denotes the pressure per unit of length of each rail necessary to depress the track (rail, tie, ballast, and roadway) one unit; for the system of units ordinarily used, it will be expressed in pounds per inch of length of rail required to depress the track 1 in. \( u \) represents the stiffness of the track, and involves conditions of tie, ballast, and roadway; it is termed the modulus of elasticity of rail support."}^{3}
\]

Definition and computation of the track modulus are very closely related as will become evident as the methods of computation are explored.
**TABLE 1**

Typical Values of the Modulus of Track Elasticity

(Selected From the First and Sixth Progress Reports of the Special Committee on Stresses in Railroad Track\(^1\))

<table>
<thead>
<tr>
<th>Rail (lb/yd)</th>
<th>Ties</th>
<th>Track and Ballast</th>
<th>u(lbs/in/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>85</td>
<td>7&quot; x 9&quot; x 8'-6&quot;</td>
<td>6&quot; fine cinder ballast</td>
<td>530</td>
</tr>
<tr>
<td></td>
<td>22&quot; c. to c.</td>
<td>Poor condition Loam and Clay subgrade</td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>7&quot; x 9&quot; x 8'-6&quot;</td>
<td>6&quot; cinder ballast</td>
<td>750</td>
</tr>
<tr>
<td></td>
<td>22&quot; c. to c.</td>
<td>Fair condition Loam and clay subgrade</td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>6&quot; x 8&quot; x 8'-0&quot;</td>
<td>6&quot; limestone</td>
<td>970</td>
</tr>
<tr>
<td></td>
<td>22&quot; c. to c.</td>
<td>Good condition before tamping Loam and clay subgrade</td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>6&quot; x 8&quot; x 8'-0&quot;</td>
<td>6&quot; limestone</td>
<td>1080</td>
</tr>
<tr>
<td></td>
<td>22&quot; c. to c.</td>
<td>After tamping Loam and clay subgrade</td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>7&quot; x 9&quot; x 8'-0&quot;</td>
<td>12&quot; limestone</td>
<td>1065</td>
</tr>
<tr>
<td></td>
<td>Good condition before tamping Loam and clay subgrade</td>
<td></td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>7&quot; x 9&quot; x 8'-0&quot;</td>
<td>12&quot; limestone</td>
<td>1090</td>
</tr>
<tr>
<td></td>
<td>After tamping Loam and clay subgrade</td>
<td></td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>7&quot; x 9&quot; x 8'-6&quot;</td>
<td>24&quot; crushed limestone</td>
<td>1200</td>
</tr>
<tr>
<td></td>
<td>22&quot; c. to c.</td>
<td>Loam and clay subgrade</td>
<td></td>
</tr>
<tr>
<td>130</td>
<td>7&quot; x 9&quot; x 8'-6&quot;</td>
<td>24&quot; gravel ballast plus 8&quot; heavy limestone</td>
<td>2900-3000</td>
</tr>
<tr>
<td></td>
<td>22&quot; c. to c.</td>
<td>Well compacted subgrade</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>7&quot; x 9&quot; x 8'-0&quot;</td>
<td>Flint gravel ballast</td>
<td>Avg. 2900</td>
</tr>
<tr>
<td></td>
<td>22&quot; c. to c.</td>
<td>Wide stable subgrade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G.E.O. fastenings</td>
<td></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>7&quot; x 9&quot; x 8'-0&quot;</td>
<td>Limestone ballast</td>
<td>Avg. 5100</td>
</tr>
<tr>
<td></td>
<td>22&quot; c. to c.</td>
<td>Wide stable subgrade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G.E.O. fastenings</td>
<td></td>
<td></td>
</tr>
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</table>
3.3 Methods of Computation

The definition of track modulus immediately gives rise to:

\[ u = \frac{p}{y} \]

where,

- \( u \) = value of track modulus
- \( p \) = uniform load per unit length of each rail
- \( y \) = track depression

This system is illustrated in Figure 1.

Translated into the more realistic case of discrete point loads or wheel loads, the track modulus may be calculated by the following relationship:

\[ u = \frac{\sum p_i}{s \sum y_i} \]

where,

- \( u \) = value of track modulus
- \( p_i \) = value of an individual wheel load
- \( y_i \) = individual tie deflection
- \( s \) = tie spacing

This method is shown in Figure 2.

A third method, based on the analysis of a beam on an elastic foundation is illustrated in Figure 3. The relationship between maximum track deflection, track modulus and rail section modulus for a continuous beam on an elastic foundation is:
\[ u \left( \frac{\text{lb}}{\text{in}^2} \right) = \frac{p \left( \frac{\text{lb}}{\text{in}} \right)}{y \left( \text{in} \right)} \]

Figure 1: Track Deflection and Track Modulus Determination for a Uniformly Distributed Rail Load
\[ u \text{ (lb/ft/in)} = \frac{\sum_{k=1}^{n} P_k}{s \cdot \sum_{j=1}^{n} y_j \text{ (in)(in)}} \]

Figure 2: Track Deflection and Track Modulus for the Track Structure under Wheel Loads
Governing Differential Equation of Beam Shape:
\[ EI y'' + u y = p(x) \]

![Diagram of beam on elastic foundation with symbols and equations]

\[ Y_0 = \frac{P}{\sqrt[3]{64 EI u^3}} \]
\[ u = \sqrt[3]{\frac{P^4}{64 EI Y_0^4}} \]

Figure 3: Track Deflection and Track Modulus for the Representation of the Track Structure as a Continuous Beam on a Continuous Elastic Foundation
\[ Y_o = - \frac{P}{4\sqrt{64EIu^3}} \]

where,

- \( Y_o \) = rail deflection under single load point
- \( P \) = magnitude of single wheel load
- \( E \) = modulus of elasticity of rail steel
- \( I \) = moment of inertia of rail section
- \( u \) = modulus of track elasticity.

To evaluate \( u \), these methods will all require field measurements of track deflections. The model developed in this investigation utilizes the material constants to compute track deflections by methods of structural analysis. Thus, the model makes possible the prediction of track modulus without previous knowledge of the deflection characteristics of a given track structure system.

3.4 Factors Influencing Track Modulus

A consideration of the four basic track structure components will illustrate the factors having an important bearing on the value of track modulus achieved.

For a purely elastic system as assumed by Talbot, the rail size (stiffness) will have no effect upon the track modulus. However, if the ballast or subgrade is weak such that permanent deformation under load may occur, the beam action of a stiff rail section may be quite significant in creating an adequate track structure.

Tie spacing, dimensions, and quality may be expected to contribute significantly to the track stiffness as reflected in the track modulus.
The quality, depth, and degree of compaction of the ballast section are important parameters defining solidarity of track construction. Subgrade quality and degree of compaction determine the strength of the foundation upon which the remainder of the track structure rests.

Important interrelationships between these factors and the track structure components will exist. Rail size will determine to some extent how much load is distributed to a particular tie. Ballast section maintenance or lack of it will determine to a large degree the pressure distribution between tie and ballast, the case of a center-bound tie being an extreme example. Weaknesses in one component of the track structure will tend to shift the burden to the remaining components.

Excellence in track construction will depend upon the selection of components which will function together to create an adequate load bearing system. Uniformity of rail support and consistency of track stiffness is of primary importance in attaining a durable, smooth riding track structure. Track maintenance programs must strive to maintain track stiffness values (track modulus) at a level somewhere near the capabilities of the given system.

The model presented attempts to take many of the factors which influence track modulus into account in determining a value of track stiffness appropriate for the system under consideration.
3.5 Relation to Track Durability

Durability of track line and surface conditions under traffic is a function of track depression under the applied loads. Track deterioration occurs at an accelerated rate as track deflections increase. The inverse relationship between track modulus and track deflection offers the conclusion that a stiff track is a stable track.

Experience with behavior of track in the field has established practical limits on track deflections under load for dependable service. Specifying track deflection is an indirect way of specifying track modulus, the former having the advantage of being much more readily interpreted in the field. Talbot's studies with the Special Committee on Stresses in Railroad Track provide the basis for the deflection limits specified in Figure 4.


Maximum Track Deflection (inches)

<table>
<thead>
<tr>
<th>Range</th>
<th>Track Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Deflection range for track which will last indefinitely.</td>
</tr>
<tr>
<td>B</td>
<td>Normal maximum desirable deflection for heavy track to give requisite combination of flexibility and stiffness.</td>
</tr>
<tr>
<td>C</td>
<td>Limit of desirable deflection for track of light construction (&lt; 100 lb).</td>
</tr>
<tr>
<td>D</td>
<td>Weak or poorly maintained track which will deteriorate quickly.</td>
</tr>
</tbody>
</table>

Values of deflection are exclusive of any looseness or play between rail and plate or plate and tie and represent deflections under load.

Figure 4: Track Deflection Criteria for Durability
Figure 5: Model System for the Simulation of the Track Structure
4. CHOICE OF MODEL

4.1 General

The objective of the simulation model developed is to create a mathematical formulation of the behavior of the track structure under load such that a determination of the expected track modulus can be made. The track deflections needed for this computation are obtained from a structural analysis procedure. Input data for the model are to be the externally applied loads and the estimated track structure parameters. A schematic representation of the model system is shown in Figure 5.

4.2 Model Possibilities

Several mathematical models were considered as a means of determining the track modulus once the parameters specifying the track structure components are delineated. The following five general model types were considered:

(1) Mechanical Analog Systems (Mass-spring combinations)
(2) Elastic Layered System Analysis
(3) Shear Layer Theory
(4) Lumped Parameter Models (Finite difference methods)
(5) Finite Element Approach

Of the five model types examined, the finite element approach offers the most comprehensive attack on the problem. The finite element method possesses the flexibility of application and the directness of approach essential to the satisfactory solution of the track structure problem.
The first three model types were found to be too restrictive for the type of analysis under consideration. A lumped parameter model was thoroughly explored but was found to present serious equation conditioning difficulties when applied to the model configuration of the track structure. A change to the finite element method of analysis eliminated this difficulty while retaining the necessary flexibility in applying boundary conditions to the system.


5. THE MODEL

5.1 Model Characteristics

The approach used to solve the problem of a theoretical determination of track modulus is of practical necessity an idealization and simplification of the actual situation. The model representation of the track structure is a two-dimensional system which considers the behavior of a longitudinal section of unit thickness along the vertical centerline of the rail. The section whose behavior is studied is shown in Figure 6.

The model accounts for the behavior of each of the four major track structure components. Rail and tie stiffness are considered along with the stiffness properties of the ballast and subgrade. There is no requirement that assumptions of homogeneous linearly elastic materials be made. The model will accept random designations of material properties and will simulate cohesionless soil behavior in tension and in shear.

Actual wheel loadings on the rail are used as input to the model system. The model takes the weight of the track materials into account.

Results available from the model include stresses and deflections at various locations throughout the model as well as the estimate of track modulus.

5.2 Model Description

The two-dimensional representation of the track structure system is analyzed through the use of the finite element method. Deflections and stresses for the system under load are obtained. The condition of plane strain is assumed valid for the section.
Figure 6: The Track Structure
To prepare the model system for analysis, the section is assumed to be fabricated out of many small square plate elements. These elements are joined at the corner points, or nodes. A typical plate element is illustrated in Figure 7(a). The shape and size of the element chosen to represent the system is arbitrary. Smaller element sizes will improve the accuracy of the representation of a continuum at the expense of increasing computational time. The model is completed by representing ties by a spring or springs of appropriate stiffness placed at a chosen spacing on the top nodes of the uppermost layer of plate elements. The rail acts as a continuous beam resting on the equivalent tie springs. Figure 7(b) shows the completed model system for the section shown in Figure 6.

Solution of the model system for deflections at the nodes and stresses in the plate elements is by stiffness matrix formulation. Equilibrium of the system is maintained by force balance at each node point. Continuity of the system is maintained by requiring that the edges of the adjacent elements always remain in contact.

The model uses an average density of the soil materials to assign loads to the node points equal to the weight of the soil mass in the immediate area of each node point. This assignment results in an approximate "hydrostatic" pressure distribution of the soil weight.

The model has the capability of accepting randomly assigned input specifications for the Young's modulus, Poisson ratio, and angle of internal shearing resistance for each element. Thus, layered systems and various soil non-uniformities may be simulated.

The particular nature of soil behavior under tensile loading and high shear stresses is represented by modifications to the solution method
(a) Plate Element

(b) Finite Element Model of Track Structure Section

Figure 7: Representation of the Track Structure by the Finite Element Model
which take account of the inability of the soil to take further loading in a particular direction. A new stiffness matrix is formulated for the changed conditions existing in the model system when shear or tensile failure of an element occurs. The Mohr fracture theory is used to determine when the shear capacity of an element is exceeded. Occurrences of shear cracking are documented by the model and appear in the printout of results. The model is thus able to behave as a non-linear system similar to actual soil behavior under high loadings.

5.3 Finite Element Representation of a Continuum

The finite element method is a generalization of standard structural analysis procedures which permits computation of the stresses and deflections in two or three dimensional structures by the same techniques which are used in the analysis of ordinary framed structures. A large scale electronic computer is an essential tool for the application of the finite element method.

The basic concept of the finite element method, and of matrix structural analysis in general, is that every structure may be considered as an assembly of a finite number of individual structural components or elements. The finite character of the structural connectivity makes analysis possible by means of simultaneous algebraic equations using matrix operations.

The finite element analysis of a continuum is divided into three basic phases:
(1) Structural idealization as a composite of finite elements

(2) Evaluation of element properties

(3) Structural analysis of the idealized system.

Once the subdivision of the original system into a finite number of discrete elements possessing given stiffness or flexibility has been accomplished, the analysis of stresses and deflections in the element assemblage under a given loading condition becomes a standard structural problem. As in any structural analysis, the essential problem is to simultaneously satisfy the following three requirements:

(1) Equilibrium; the internal element forces acting at each node point must be equal to the externally applied nodal force.

(2) Compatibility; the element deformations must be such that they continue to meet at the node points in the loaded condition.

(3) Force-deflection relationship; the internal forces and internal displacements within each element must be related as required by its individual geometric and material property characteristics.

In order that the finite element idealization may provide a reasonable representation of the actual continuum, each element must be required to deform in a manner similar to the deformations developed in the corresponding region of the continuum. A basic and critical operation in the definition of an element stiffness matrix is the choice of the deformation characteristics to be allowed. The deformations of adjacent elements must be compatible.
The use of completely compatible element deformation patterns is an important factor in establishing the reliability of a finite element analysis. For the case of square plate stress elements, the specification that internal displacements vary linearly in two perpendicular directions will ensure that any straight line in the undeformed element will remain straight during deformation. As a result, all of the straight line element boundaries will remain in contact as the elements distort, and the nodal compatibility established by the displacement analysis will ensure complete boundary compatibility between node points as well as at node points.

5.4 Displacement Method of Structural Analysis

Six basic steps comprise the operations performed in a displacement method analysis of any structural system:

1. Evaluation of the stiffness properties of the individual structural elements, expressed in any convenient local (element) coordinate system.

2. Transformation of the element stiffness matrix from the local coordinate system to a form relating to the global coordinate system of the complete structure.

3. Superposition of the individual element stiffnesses contributing to each node point to obtain the total nodal stiffness matrix \([K]\).

4. Formulation and solution of the equilibrium equations expressing the relationship between the applied nodal forces \([R]\) and the resulting nodal displacements \(\{r\}):

\[
\{R\} = [K] \{r\} .
\]

5. Evaluation of the element deformations from the completed nodal displacements by kinematic relationships.

6. Determination of element forces from the element deformations by means of the element stiffness matrices.
5.5 Finite Element Stiffness Analysis

The standard finite element stiffness analysis consists of seven procedural steps (the following notation is for a two-dimensional element):

(1) Express internal displacements, \( v \), in terms of displacement functions, \( M \), as related to the generalized coordinates, \( \alpha \);

\[
\{v(x,y)\} = [M(x,y)] \{\alpha\}
\]

(2) Evaluate nodal displacement components, \( v_i \), in terms of the generalized coordinates;

\[
\{v_i\} = [A] \{\alpha\}
\]

Matrix \([A]\) is obtained from substitution of the coordinates of the node points into the displacement function matrix \([M]\).

(3) Express the generalized coordinates in terms of the nodal displacements;

\[
\{\alpha\} = [A^{-1}] \{v_i\}
\]

(4) Evaluate the element strains, \( \varepsilon \);

\[
\{\varepsilon(x,y)\} = [B(x,y)] \{\alpha\}
\]

Matrix \([B]\) is obtained from \([M]\) by appropriate differentiation of the displacement functions.
(5) Evaluate the element stresses, $\sigma$;

$\{\sigma(x,y)\} = [D] \{\varepsilon(x,y)\} = [D] [B(x,y)] \{\alpha\}$

The stress-strain matrix, $[D]$, represents the specific elastic characteristics of the finite element material.

(6) Compute the generalized coordinate stiffness of the element, $\bar{k}$.

By the principle of virtual displacements, the internal virtual work, $dW_I$, of any differential volume $dV$ of the element is given by the product of the actual stresses $\sigma$ and the virtual strains $\varepsilon$ through which they move;

$$dW_I = \varepsilon^T \sigma \, dV$$

or using the results of steps (4) and (5);

$$dW_I = \alpha^T B^T D B \alpha \, dV$$

where $\alpha$ represents the virtual generalized coordinate displacements associated with $\varepsilon$.

The total internal virtual work $W_I$ is obtained by integrating the latest equation over the volume of the element;

$$W_I = \alpha^T \left[ \int_{\text{vol}} B^T D B \, dV \right] \alpha$$

The external work associated with the virtual generalized displacements $\bar{\alpha}$ is given by;

$$W_E = \bar{\alpha}^T \beta$$
where \( \beta \) represents the generalized forces corresponding with displacements \( \alpha \). For unit values of the virtual displacement components, the equating of external to internal virtual work yields:

\[
\beta = \left[ \int_{\text{vol}} B^T D B \, dv \right] \alpha
\]

Thus the generalized coordinate stiffness of the element is:

\[
\bar{k} = \int_{\text{vol}} B^T D B \, dV
\]

(7) Transform to the desired nodal point stiffness, \( k \), by a standard coordinate transformation:

\[
k = [A^{-1}]^T [\bar{k}] [A^{-1}]
\]

The track structure model has been formulated as an assemblage of square plate elements of unit thickness. By defining the parameters:

- \( \phi \) = angle of internal friction
- \( E \) = Young's modulus
- \( \mu \) = Poisson's ratio
- \( a \) = element length
- \( \eta \) = element coordinate \( 0 \leq \eta \leq 1 \)
- \( \xi \) = element coordinate \( 0 \leq \xi \leq 1 \);

the basic matrices for the evaluation of the stiffness of the element during elastic behavior are:
\[
D = \frac{E}{(1 + \mu) (1 - 2\mu)} \begin{bmatrix}
1-\mu & \mu & 0 \\
\mu & 1-\mu & 0 \\
0 & 0 & \frac{1-2\mu}{\mu}
\end{bmatrix}
\]

\[
B = \frac{1}{a} \begin{bmatrix}
-(1-\eta) & 0 & -\eta & 0 & \eta & 0 & (1-\eta) & 0 \\
0 & -(1-\xi) & 0 & (1-\xi) & 0 & \xi & 0 & -\xi \\
-(1-\xi) & -(1-\eta) & (1-\xi) & -\eta & \xi & \eta & -\xi & (1-\eta)
\end{bmatrix}
\]

The plate stiffness matrix under elastic behavior, \( k_E \), is represented as:

\[
k_E = \int_0^1 \int_0^1 (B^T D B) \, d\eta \, d\xi
\]

For the case where shear or tensile cracking of the element occurs, the crack direction coordinate transformation matrix is expressed in terms of direction cosines as:

\[
R = \begin{bmatrix}
\cos^2 \gamma + \sin^2 \gamma + \frac{1}{2} \sin 2\gamma \\
\sin^2 \gamma + \cos^2 \gamma - \frac{1}{2} \sin 2\gamma \\
1^2 & m^2 & 1m \\
-m^2 & 1^2 & -1m
\end{bmatrix}
\]
Element behavioral characteristics are expressed by the matrix $[\bar{H}]$:

For tensile cracking:

$$\overline{H}_T = \begin{bmatrix} C_1 & 0 \\ 0 & 0 \end{bmatrix}$$

For shear cracking:

$$\overline{H}_S = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}$$

where,

$$C_1 = \frac{E(1 + \mu)}{(1 + \mu)(1 - 2\mu)}$$

$$C_2 = \frac{E \mu}{(1 + \mu)(1 - 2\mu)}$$

$$C_3 = m C_1$$

$$C_4 = m C_2$$

For $m = \frac{1 - \sin \phi}{1 + \sin \phi}$. 

The plate stiffness matrix under tensile cracking, $k_T$, becomes:

$$k_T = \int_{0}^{1} \int_{0}^{1} (B^T \bar{R}^T \overline{H}_T \bar{R} B) \, d\eta \, d\xi$$
and \( k_s \) is the plate stiffness matrix under shear cracking occurrence:

\[
k_s = \int_0^1 \int_0^1 (b^T R^T H_s R b) \, dn \, d\xi.
\]

With the formation of the plate stiffness matrices, the solution of the model system proceeds using the basic stiffness relationship:

\[
\beta = k \alpha.
\]

For the track structure problem the upper elements of the model are changed to represent the rail and tie behavior. The stiffness matrix of the upper elements is therefore formulated to give results in terms of rail moments and tie forces at the two upper nodes of the element. This stiffness matrix has the following form:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & K_1 & 0 & -K_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 4c_1L^2 & 6c_1L & 2c_1L^2 & -6c_1L & 0 & 0 \\
0 & -K_1 & 6c_1L & K_1 + \frac{c_1}{12c_1} & 6c_1L & -12c_1 & 0 & 0 \\
0 & 0 & 2c_1L^2 & 6c_1L & 4c_1L^2 & -6c_1L & 0 & 0 \\
0 & 0 & -6c_1L & -12c_1 & -6c_1L & K_2 + \frac{c_2}{12c_1} & 0 & -K_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & K_2 & 0 & K_2
\end{bmatrix}
\]
where,
\[ K_1 = \text{equivalent spring constant of tie at left node} \]
\[ K_2 = \text{equivalent spring constant of tie at right node} \]
\[ L = \text{length of element} \]
\[ C_1 = \frac{E}{L^3} \]

\( E = \text{modulus of elasticity of rail steel} \)
\( I = \text{moment of inertia of the rail section.} \)

A short discussion of the boundary conditions established for a finite section of the track structure will serve to complete the discussion of the model.

5.6 Boundary Conditions

The boundary conditions for the model have been chosen in an attempt to simulate track behavior as closely as possible. The surface boundary is free to move in either coordinate direction subject to the constraints of the applied load system. A fixed condition was chosen to represent the lower boundary of the model. The contribution to surface deflection at considerable depths is small.

The model will accept any designation for the boundary condition of the sides. Any node on this boundary may be specified to be fixed or to be free to move in either the vertical or the horizontal direction. For the track structure system a boundary free in the vertical direction but fixed in the horizontal direction was chosen. The node representing the rail end may be designated as fixed or free in the vertical coordinate direction, and may have either zero or full moment restraint at the boundary.

The choice of boundary constraints and the options made available allow assignment of conditions to create a reasonable simulation of actuality.
6. METHOD OF SOLUTION

Once the model is represented in terms of finite elements and the behavior of each element is described in mathematical terms, suitable computational methods are sought for the general problem.

The concepts of the theory of matrix structural analysis were utilized to prepare a computer program for the solution of the model system. The Fortran IV language was used in generating a solution routine for the IBM System 360/75. The program is self-sufficient, requiring only the basic library subroutines for support. In its present form the program utilizes only the core storage capacity of the machine. A general flow diagram of the solution procedure is given in Figure 8. The program listing is reproduced in the Appendix.

In the process of solution, average stresses in the elements are computed and the value is assigned to the center point of the element. Deflections in both coordinate directions are calculated for each node point.

During the solution of the model system, the program tabulates those elements which encounter shear or tensile stress in excess of their capacity. On the following iterative solution of the system, these particular elements are assumed to have failed, and the load is redistributed to neighboring elements. The iterative solution process is performed a specified number of times, currently five cycles. If the loading on an element changes from a tensile failure condition to a state which would not involve failure, then that particular element is reinstated as a load carrying component of the system. The choice of the number of iterations used to attain a solution will be a function of the system analyzed. A construction involving weak materials will be expected to yield slow convergence. Savings in computation
Figure 8: Flow Diagram for the Computer Program Solution of the Model System
time may be effected for very stiff materials where convergence is rapid. This combination of shear and tensile failure criteria with the iterative solution technique allows the model solution to simulate the non-linear behavior of soil materials under heavy loadings.
7. APPLICABILITY OF RESULTS

7.1 General

Model accuracy was checked by two approaches. A series of comparisons with the results obtained by previous investigations into the track modulus or related problems were made. The second approach was to check several model representations of the same system against each other.

7.2 Effect of Boundary Conditions

The effects on accuracy of the assignment of a fixed lower boundary to the model system is of some interest. Computations of the deflections directly under a loaded tie for the case of a homogeneous elastic subgrade were performed utilizing Newmark's Charts. This analysis showed that 91 percent of the total surface deflection occurred within a depth of 44 inches (the thickness of soil materials used in the example illustrating the application of the model, Section 8.1). When non-linear behavior of the materials is allowed, as in the case of shear or tensile cracking of some of the elements, the effect of neglecting the elastic deformation of the deeper layers is decreased. The added deflection due to non-linear behavior will occur in the uppermost layers of the system in regions of high stress. The behavior of the deeper layers, at lower stress levels, will remain elastic. With elastic deflections now comprising a part of the total deflection, the relative error of neglecting a small part of the total elastic deformation is reduced. To illustrate, the example problem of section 8.1 has an elastic deflection error of 0.0032 inches. This error is about 1.5 percent of the final deflection after five iterations of 0.2079 inches.
The error involved in assuming a fixed ended boundary condition for the rail is negligible. Under the applied load, the rail would deflect 10.5 inches without ballast support. Thus, an insignificant part of the load will be carried by the fixed boundary; essentially the full load is applied to the tie-ballast-subgrade structure.

7.3 Comparison with Previous Results

A comparison of the results of the stress distribution predicted by the model and those predicted by the Boussinesq distribution is of interest. The model was used to obtain values of elastic stresses under the action of a strip load. Contours of equal vertical stress were prepared for both the Boussinesq analysis and the model solution. These results are shown in Figures 9 and 10 and a comparison is given in Figure 11.

Close agreement between the two cases was found for the contours with vertical stress more than ten percent of the surcharge load. Some deviation from the ideal case was found in the contours representing low values of vertical stress at distances remote from the loaded area. Since the major portion of the deflection occurs in the highly stressed zone directly under the load, the model is judged to be sufficiently accurate for the purpose of calculating track deflections on which to base an estimate of the track modulus.

A comparison of results obtained from the model system with laboratory determined pressure distributions in ballast was made. The experimental results were those obtained from the work of Salem and Hay at the University of Illinois. A plot of the results of this comparison is shown in Figure 12. No model results for the light loading are presented.
Figure 9: Boussinesq Stress Distribution for a Strip Load
Figure 10: Strip Load Stress Distribution for the Finite Element Model (4 inch element size)
Figure 11: A Comparison of the Stress Distributions Obtained by the Model and the Boussinesq Theory
Figure 12: A Comparison of the Vertical Pressure Distributions Obtained by the Model and by the Laboratory Determinations of Salem and Hay
The model system currently experiences instability problems in cases of very light loadings applied to stiff systems. Further work in this area is desirable.

Although rough estimates of the material properties used in the experimental study had to be used, the agreement between the two sets of results is reasonably good, with the model simulation tending to give higher ballast pressures than those measured experimentally, particularly at the greater depths. This may be explained in part by the use of sliding boundary conditions in the model solution as an approximation of the side friction effect occurring in the test ballast box. The model appears to be an acceptable approximation of the pressure relationship existing in ballast under tie loads.

A comparison of the deflections obtained from the analysis of a continuous beam on a continuous elastic foundation (Talbot method of analysis) and the deflection obtained from a linearly elastic solution by the model is shown in Figure 13. The solutions are both based on an elastic track modulus of 2707 lb/in/in. Agreement between the two is good. However, the model system was based on very low strength materials in order to obtain this value of track modulus. Actual material stiffnesses are approximately four times the values used for this comparison. The introduction of shear and tensile cracking phenomena into the model allowed simulation with reasonable material constants while maintaining realistic track deflection and track modulus values.

In all, the model can be accepted as a reasonable simulation of the pressure-deflection relationships existing in a loaded track structure.
Figure 13: Track Deflections: Model Elastic Solution and the Talbot Solution for $u = 2707$ lb/in/in
7.4 Consideration of Element Size

A study of the effect of the size of the plate elements on the results was made by representing the same model system by elements of 2 inch, 4 inch and 8 inch size. Smaller size of elements will improve the accuracy of the stress determination while larger element sizes will allow solution of a larger section of the track structure for a given computation time and machine storage allotment.

Reasonable agreement exists between the values obtained from the elastic solution of the 3 cases as shown in Table 2. The term elastic solution is used in this report to designate the initial solution of the system obtained by the model before cracking effects are accounted for. In this state the model behaves purely elastically. The method of stress calculation used assigns stress averages to the centers of the elements. The method of applying external loads for the ties results in large loads being applied to a few node points. This combination renders the smaller 2 inch model particularly susceptible to shear and tensile failure. Further reduction in grid size would improve the loading situation at the expense of the size of system which could be simulated.

The 4 inch and 8 inch element sizes yield much the same values for both initial and final values of track modulus and deflection. The maximum stresses for the 8 inch model are about one-half the maximum stresses for the 4 inch model as would be expected from the stress assignment method used.

A 4 inch element size was selected for the simulation of the track structure as giving a model system relatively sensitive to stresses while allowing an adequate size of the track structure to be simulated. The 2 inch element size was both hypersensitive to the loading system imposed and
<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>2in</td>
<td>68478</td>
<td>6631</td>
<td>0.00844</td>
<td>101376</td>
<td>34.6</td>
</tr>
<tr>
<td>4</td>
<td>75407</td>
<td>26737</td>
<td>0.0078</td>
<td>97955</td>
<td>19.6</td>
</tr>
<tr>
<td>8</td>
<td>66404</td>
<td>29335</td>
<td>0.0069</td>
<td>105027</td>
<td>12.0</td>
</tr>
</tbody>
</table>
severely limiting to the size of system which could be analyzed. Since the
program is designed to use only the core storage facilities of the machine,
there are limitations on the number of elements which can be handled. Sizes
below a 4 inch element size would cause the boundaries of the model to have
an appreciable effect on the results. The 8 inch size is acceptable,
although some sacrifice in the accuracy of stress determinations must be
made.

On the whole, the 4 inch element size provides a practical system
for the analysis of track behavior, offering computing economy with reasonable
accuracy.


8. APPLICATION OF THE MODEL TO THE TRACK STRUCTURE

8.1 Example Application

As an example problem, the model was used to study the effect of a single 30,000 pound wheel load on a moderately stiff track structure. A 115 pound RE section rail on oak ties resting on 24 inches of ballast over a stable subgrade was chosen.

A 4 inch element size was used. The system simulated was 13 nodes (12 elements) deep by 65 nodes (64 elements) wide with advantage being taken of symmetry about the vertical axis. This made a model of a track section 44 inches deep by 504 inches long (3.67 feet by 42 feet) with a wheel load applied on the rail at the line of symmetry. The 115 lb. rail has a moment of inertia of 65.6 in\(^4\). The ties; 8.5 feet long, 8 inches wide, 8 inches thick at 20 inch spacing, were assigned a spring constant of 1,750,000 psi. The tie load factor, an estimate of the percentage of uniform tie pressure acting directly under the rail, was assigned a value of 1.20. The 24 inch thick layer of ballast was assumed to have a modulus of elasticity of 20,000 psi, a Poisson ratio of 0.32, with an angle of internal friction of 45 degrees. The remaining 20 inches of depth of the model was assumed to be a compacted sand with corresponding material properties of 10,000 psi, 0.34, and 40 degrees. Average weight of the soil materials was taken as 110 pounds per cubic foot.

Loads representative of rail weight, track fastenings and a portion of the tie weight were applied at the locations of the ties. This assignment improves the accuracy of the modeling and improves the deflection behavior of the rail; relatively large positive deflections of the rail outside the region
of the wheel load will result if it is assumed weightless.

The boundary conditions assigned to the model attempt to duplicate those existing in the field. The ballast and subgrade are assumed constrained horizontally at the side boundaries but are allowed to move vertically freely. The end of the rail is simulated as being fixed in the vertical direction and held in a moment clamp. This approximates the effect of the continuity of the rail which exists beyond the model boundary. The model system is illustrated in Figure 14.

The initial elastic solution of the system assigns a track modulus of 18232 lb/in/in to the structure with a maximum deflection under the load of 0.0326 inches. The term elastic solution is used in this report to designate the solution obtained by the first iteration of the model, before cracking has occurred. The results of further iterations on the system as it adjusts to tensile and shear constraints are given below:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Track Modulus</th>
<th>Maximum Rail Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>18232 lb/in/in</td>
<td>0.0326 in.</td>
</tr>
<tr>
<td>2nd</td>
<td>8781</td>
<td>0.0428</td>
</tr>
<tr>
<td>3rd</td>
<td>4233</td>
<td>0.1362</td>
</tr>
<tr>
<td>4th</td>
<td>4088</td>
<td>0.1865</td>
</tr>
<tr>
<td>5th</td>
<td>2202</td>
<td>0.2079</td>
</tr>
<tr>
<td>6th</td>
<td>3542</td>
<td>0.1972</td>
</tr>
<tr>
<td>7th</td>
<td>3520</td>
<td>0.1963</td>
</tr>
</tbody>
</table>

At the fifth iteration the solution is converging and the system is stabilizing under the applied load. Some additional shear and tensile cracking occurs between the 6th and 7th iterations. The convergence of the
Figure 14: The Model Representation of the Track Structure for the Example Problem
maximum rail deflections is not matched by a similar convergence in the track modulus values. The reason is that the track modulus parameter is very sensitive to the degree to which the rail deflects in an upward direction beyond the immediate region of the wheel load.

The input data and a partial output listing are shown in Tables 3 and 4.

The deflected shape of both the initial elastic solution and the final deflected shape are shown in Figure 15. The solution for the case of a continuous beam on an elastic foundation of modulus 2202 lb/in/in is also shown.

The effect of the assumption made in the Talbot analysis of assuming that negative pressures can develop is evident in the comparison made in Figure 15. The finite element model makes no such assumption. After the first trial, the only force holding the rail down is its own weight and that of the rail fastenings. This effect is also shown in the rail moment plot of Figure 16. In the case of the model solution the rail must provide its own hold down force by its own weight and by moment restraint at the boundary as no negative foundation pressures are allowed to develop.

The stresses derived from the solution of systems with large element sizes can not be depended upon to produce reliable stress distributions, but may be used as an indication of the pressure distributions occurring in the ballast and soil materials. A stress contour plot of the element stresses in the vicinity of the load is shown in Figure 17. An averaging of the four neighboring element stresses results in the stress distribution shown in Figure 18.

It is important to caution again that these distributions are based on average stresses calculated for the centers of 4 inch elements.
# TABLE 3

**Input Data for the Example Problem**

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**TRACK MODULUS - ECLIPSE PERTIC**

2202.

**TRACK MODULUS - TIE EFFECTIVE**
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**Shear Crack Formed**

**Shear Crack Directivas in Stress Point 758**
- ARE -62.86 and -107.86 CC FROM HORIZONTAL

**Crack Direction in Stress Point 758**
- IS -85.26 CC FROM HORIZONTAL

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**Shear Crack Formed**

**Shear Crack Directivas in Stress Point 760**
- ARE -63.58 and -108.58 CC FROM HORIZONTAL

**Crack Direction in Stress Point 760**
- IS -87.44 CC FROM HORIZONTAL

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**Shear Crack Formed**

**Shear Crack Directivas in Stress Point 761**
- ARE -44.94 and -105.94 CC FROM HORIZONTAL

**Crack Direction in Stress Point 761**
- IS -87.44 CC FROM HORIZONTAL

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**Shear Crack Formed**

**Shear Crack Directivas in Stress Point 762**
- ARE -44.94 and -105.94 CC FROM HORIZONTAL

**Crack Direction in Stress Point 762**
- IS -87.44 CC FROM HORIZONTAL

<table>
<thead>
<tr>
<th>NODE</th>
<th>STRESS X</th>
<th>STRESS Y</th>
<th>SHEAR STRESS</th>
<th>PRINCIPAL STRESSES x AND y</th>
</tr>
</thead>
<tbody>
<tr>
<td>762</td>
<td>-0.3419459E 01</td>
<td>-0.1522713E 02</td>
<td>-0.4208523E 01</td>
<td>-0.1016665E 01</td>
</tr>
</tbody>
</table>

**Shear Crack Formed**

**Shear Crack Directivas in Stress Point 763**
- ARE -65.77 and -110.77 CC FROM HORIZONTAL

**Crack Direction in Stress Point 763**
- IS -88.27 CC FROM HORIZONTAL

<table>
<thead>
<tr>
<th>NODE</th>
<th>STRESS X</th>
<th>STRESS Y</th>
<th>SHEAR STRESS</th>
<th>PRINCIPAL STRESSES x AND y</th>
</tr>
</thead>
<tbody>
<tr>
<td>763</td>
<td>-0.4254688E 01</td>
<td>-0.1266105E 02</td>
<td>-0.2259452E 01</td>
<td>-0.2343526E 01</td>
</tr>
</tbody>
</table>

**Shear Crack Formed**

**Shear Crack Directivas in Stress Point 764**
- ARE -43.49 and -113.49 CC FROM HORIZONTAL

**Crack Direction in Stress Point 764**
- IS -88.45 CC FROM HORIZONTAL

<table>
<thead>
<tr>
<th>NODE</th>
<th>STRESS X</th>
<th>STRESS Y</th>
<th>SHEAR STRESS</th>
<th>PRINCIPAL STRESSES x AND y</th>
</tr>
</thead>
<tbody>
<tr>
<td>764</td>
<td>-0.4475122E 01</td>
<td>-0.1172265E 02</td>
<td>-0.2154863E 01</td>
<td>-0.2307969E 01</td>
</tr>
</tbody>
</table>

**Shear Crack Formed**

**Shear Crack Directivas in Stress Point 765**
- ARE -63.51 and -113.51 CC FROM HORIZONTAL

**Crack Direction in Stress Point 765**
- IS -88.51 CC FROM HORIZONTAL

<table>
<thead>
<tr>
<th>NODE</th>
<th>STRESS X</th>
<th>STRESS Y</th>
<th>SHEAR STRESS</th>
<th>PRINCIPAL STRESSES x AND y</th>
</tr>
</thead>
<tbody>
<tr>
<td>765</td>
<td>-0.4585532E 01</td>
<td>-0.1166349E 02</td>
<td>-0.2394774E 01</td>
<td>-0.2433434E 01</td>
</tr>
<tr>
<td>766</td>
<td>-0.6277611E 01</td>
<td>-0.1296732E 02</td>
<td>-0.2186323E 01</td>
<td>-0.2359939E 01</td>
</tr>
<tr>
<td>767</td>
<td>-0.6754925E 01</td>
<td>-0.1192528E 02</td>
<td>-0.2372152E 01</td>
<td>-0.3833270E 01</td>
</tr>
</tbody>
</table>
Figure 15: Comparison of Rail Deflections: Model Behavior and Beam on an Elastic Foundation Analysis for the Example Problem
Figure 16: Comparison of Maximum Rail Moments: Model Behavior and Beam on an Elastic Foundation Analysis for the Example Problem
Figure 17: Stress Contours from the Model Solution of the Example Problem

Depth in Inches
Figure 18: Stress Contours from the Model Solution of the Example Problem Averaged for the Four Adjacent Elements.
and thus give only a crude approximation of the true stress distribution which exists in the actual track structure.

8.2 Other Applications

The model representation of the track structure is quite general and possesses a relatively high degree of flexibility. These features allow the model to be effectively applied to the practical study of the track structure and investigations into the effects of various track material parameters on track stiffness. The model may be used directly to study the effects of any of the following conditions on track stiffness behavior:

1. rail size; (beam stiffness)
2. ties;
   a. stiffness (species)
   b. length
   c. spacing
   d. effect of poor spacing
   e. effect of weak or partially decayed ties
   f. effect of missing or broken ties
   g. effect of assumed load distributions (tie load factor)
3. ballast;
   a. type (varied E, μ, φ; individual or multiple parameter variation)
   b. depth (lift thickness)
4. subgrade;
   a. type (varied E, μ, φ; individual or multiple parameter variation)
   b. depth (as related to ballast thickness or model size)
5. other effects;
   a. soft spots or weak pockets in ballast and subgrade
   b. maintenance (compare track stiffness based on material properties before and after maintenance work)
   c. changed wheel loadings (configurations and magnitudes)
   d. multi-layered systems
   e. random variation of E, μ, φ, throughout model system
   f. composite track structures (bituminous or other stabilized layers)

Some usual ranges of several of the input parameters are listed in Table 5.
<table>
<thead>
<tr>
<th>(1) Rail:</th>
<th>90 lb RA</th>
<th>$I = 38.7 \text{ in}^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 lb RE</td>
<td>$I = 49.0$</td>
</tr>
<tr>
<td></td>
<td>115 lb RE</td>
<td>$I = 65.6$</td>
</tr>
<tr>
<td></td>
<td>132 lb RE</td>
<td>$I = 88.2$</td>
</tr>
</tbody>
</table>

(2) Ties: Modulus of elasticity in compression perpendicular to the grain:

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>White Oak</td>
<td>1,750,000 psi</td>
</tr>
<tr>
<td>Red Oak</td>
<td>2,010,000 psi</td>
</tr>
<tr>
<td>Southern Pine</td>
<td>2,090,000 psi</td>
</tr>
<tr>
<td>Douglas Fir</td>
<td>1,830,000 psi</td>
</tr>
<tr>
<td>Gum</td>
<td>1,800,000 psi</td>
</tr>
</tbody>
</table>

(3) Ballast and Subgrade Materials:

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ (psi)</th>
<th>$\mu$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crushed rock</td>
<td>35000 - 40000</td>
<td>0.30</td>
<td>50</td>
</tr>
<tr>
<td>Dense sand and gravel</td>
<td>15000 - 30000</td>
<td>0.30 - 0.36</td>
<td>38 - 45</td>
</tr>
<tr>
<td>Dense sand</td>
<td>7500 - 12000</td>
<td>0.30 - 0.35</td>
<td>35 - 42</td>
</tr>
<tr>
<td>Loose sand</td>
<td>1500 - 3000</td>
<td>0.30 - 0.32</td>
<td>28 - 33</td>
</tr>
<tr>
<td>Clay, semisolid</td>
<td>1000 - 2000</td>
<td>0.35 - 0.40</td>
<td>30 - 32</td>
</tr>
<tr>
<td>Clay, stiff plastic</td>
<td>600 - 1200</td>
<td>0.40 - 0.45</td>
<td>28 - 30</td>
</tr>
</tbody>
</table>

(4) Densities of Soil Materials:

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (pcf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crushed slag</td>
<td>75 - 85</td>
</tr>
<tr>
<td>Crushed limestone</td>
<td>95 - 105</td>
</tr>
<tr>
<td>Crushed granite</td>
<td>115 - 120</td>
</tr>
<tr>
<td>Average subgrade materials</td>
<td>110 - 120</td>
</tr>
</tbody>
</table>
A few of the possibilities have been tried. The model system was solved for several rail size sections. The results of the comparison of track deflection, track modulus, and rail moments are given in Figures 19, 20, and 21.

The influence of ballast modulus, ballast depth, and subgrade modulus on the behavior of the example problem system were also examined. The results are shown in Figures 22 through 30.

A trial investigation of a ballast section based on the example model system with a hemispherical soft spot of subgrade material having an approximate diameter of 18 inches directly under the load and extending upward into the ballast showed a reduction of the track modulus to 1561 lb/in/in with the maximum rail deflection increasing to 0.2340 inches and rail moment increasing to 325,376 in-lbs.

The example model system was used to study the effect of a missing or weak tie under the wheel load. For the case of a weak tie (one-half normal tie stiffness) deflection under the load was 0.1850 inches, a decrease, while modulus and rail moment values remained much the same at 2211 lb/in/in and 277,000 in-lbs respectively.

When the tie under the load was removed the modulus dropped to 1236 lb/in/in, deflection increased to 0.3600 inches and rail moment increased to 469,000 in-lbs.

The case of the weak tie under the load results in a very favorable track behavior situation for the static load, i.e. the deflection is less than the case where tie stiffness are the same. This results from the improved load distribution between the weak tie and its neighboring ties and the consequent more favorable load distribution to the ballast, as compared to
Figure 19: The Effect of Rail Size on Maximum Track Deflection as Determined by the Model.
Figure 20: The Effect of Rail Size on Track Modulus as Determined by the Model.
Figure 21: The Effect of Rail Size on Maximum Rail Moment as Determined by the Model
Figure 22: The Effect of Ballast Modulus on Maximum Track Deflection as Determined by the Model.
Figure 23: The Effect of Ballast Modulus on Track Modulus as Determined by the Model
Figure 24: The Effect of Ballast Modulus on Maximum Rail Moment as Determined by the Model
Figure 25: The Effect of Ballast Depth on Maximum Track Deflection as Determined by the Model
Figure 26: The Effect of Ballast Depth on Track Modulus as Determined by the Model
Figure 27: The Effect of Ballast Depth on Maximum Rail Moment as Determined by the Model
Figure 28: The Effect of Subgrade Modulus on Maximum Track Deflection as Determined by the Model
Figure 29: The Effect of Subgrade Modulus on Track Modulus as Determined by the Model
Figure 30: The Effect of Subgrade Modulus on Maximum Rail Deflection as Determined by the Model
the system with equal tie stiffnesses. This effect is not present if the tie under the load is removed. Obviously, if moving wheel loads are considered, this beneficial effect will not be maintained.

Another simulation involved assuming the loaded tie to be poorly spaced at 24 inches center to center from its neighbors. This resulted in the modulus dropping to 1967 lb/in/in with the deflection increasing to 0.3116 inches and maximum rail moment increasing to 367,000 in-lbs.

The results of the runs for tie and soft spot variations of the example problem are summarized in Table 6.


<table>
<thead>
<tr>
<th>Configuration</th>
<th>Track Modulus (lb/in/in/in)</th>
<th>Maximum Rail Moment (in-lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example problem without variation</td>
<td>0.2079</td>
<td>299,000</td>
</tr>
<tr>
<td>Weak tie under load</td>
<td>0.1850</td>
<td>277,000</td>
</tr>
<tr>
<td>Missing tie under load</td>
<td>0.3600</td>
<td>469,000</td>
</tr>
<tr>
<td>Poorly spaced tie under load</td>
<td>0.3116</td>
<td>367,000</td>
</tr>
<tr>
<td>Soft spot in ballast</td>
<td>0.2340</td>
<td>325,000</td>
</tr>
</tbody>
</table>

Summary of Model Behavior for Tie and Soft Spot Variations
9. SUMMARY AND CONCLUSIONS

The primary purpose of this research was to develop a method of evaluation of the track stiffness properties under static loadings. The two-dimensional track structure model presented fulfills this intended purpose. An adequate means of determining the strains and stresses and in turn deflections and track modulus has been developed. The wide range of track structure parameters considered in the model provides a general approach to the problem which possesses sufficient flexibility to be of value in track structure studies. Comparisons with previous results have shown that the model is capable of producing reliable and reasonably accurate results.

The model is not entirely self-sufficient. Values for the physical properties of the soil materials used in the track structure must be supplied as input data. Unfortunately, very little is known about the characteristics of ballast and subgrade materials as they exist in the track structure. Even less information as to the nature of the variation in these materials is available. The caution that the model solution can be no better than the data supplied must be mentioned.

A comparison between the model solution and the Boussinesq analysis has been prepared and is presented in Figures 9, 10 and 11.

A comparison of the deflections of a purely elastic model system with the results from the beam on an elastic foundation analysis are presented in Figure 13. This comparison is based on a model system having much lower than normal soil property values. If non-linear behavior of this system were allowed, track behavior would deviate considerably from the elastic assumption.
An example problem of an analysis of a track structure using the model was presented in Section 8. The behavior of the system is summarized in Figures 15 through 18.

The comparisons of deflections in Figure 15 and rail moments in Figure 16 show significant differences in track behavior. The nature of the deflection and pressure distributions between the two cases will account for the difference in the results. The simulation model makes no assumptions about the development of negative foundation pressures or linear elastic soil behavior.

Despite the discrepancies in the deflection and pressure distributions, both the model and the Talbot analysis agree very closely on the maximum rail moment developed under a wheel load. This result tends to verify the value of the beam on elastic foundation analysis in computations of rail moments. Maximum rail stresses predicted by either method would not differ significantly.

The comparisons presented in Figures 19 through 30 illustrate the effect of rail size, ballast stiffness, ballast depth and subgrade stiffness on track structure behavior as determined by maximum rail deflection, track modulus and maximum rail moment.

The plots of moment and track modulus generally display significant trends.

Rail size has a considerable influence on track behavior.

The approximate stress plots of Figures 17 and 18 show that stresses in the ballast tend to become constant at a depth of 22 to 24 inches below the base of tie for the system under study. This would indicate that the ballast thickness should be at least this thick in order to prevent non-uniform stresses on the softer subgrade with their attendant problems of
ballast pockets, pumping and rough track.

This study and the model system developed raise the important question of track modulus definition. The value of track modulus defined by the beam on an elastic foundation analysis may not have direct significance in the behavior of actual track other than in the prediction of bending moments. The upward deflection of the rail in a conventional track structure may not play the role assigned to it by the equation defining track modulus. Perhaps the consideration of downward deflections alone under a specified standard loading would provide a better standard of comparison between varying track structures. The use of track modulus to describe track stiffness deserves careful study. Perhaps a more meaningful parameter or method of determination of track stiffness is possible.

In some cases the convergence of the model solution is not clear. Unusual combinations of track structure components or material properties will often result in excessive deflections, negative track moduli or oscillatory rather than convergent solutions. Some additional work is required on the model in order to allow it to handle such cases.
10. FUTURE DEVELOPMENT

The model developed for the analysis of the track structure by this study remains in an elementary stage of development.

More complete data on track material constants will improve the usefulness of the model in simulating field conditions. Further refinement of the finite element representation would include graduated element sizes with the smaller element sizes in the regions of high stress to improve accuracy. Information on the statistical variation of the material properties when in place in the roadbed could be utilized in a modification to the program which would allow controlled random assignment of material properties to the various elements in each of several layers. Provision for the inclusion of soil cohesion in the shear and tensile failure criteria would allow simulation of cohesive subgrade materials.

Improved representation of boundary conditions is possible. A state of partial fixity could be simulated. A more realistic representation of the rail boundary condition would involve simulation of partial fixity.

The possibility of larger capacity computing machines raises the hope that a three-dimensional simulation of the track structure system may be not too far in the future.

The extension of track structure models into the realm of track behavior under dynamic loadings is a challenging goal.
BIBLIOGRAPHY


APPENDIX

PROGRAM LISTING FOR THE SIMULATION MODEL OF BALLAST SUPPORT AND THE MODULUS OF TRACK ELASTICITY
C
C******************************************************************************
C* A SIMULATION MODEL OF BALLAST SUPPORT AND THE MODULUS OF
C* TRACK ELASTICITY
C******************************************************************************
C
C******************************************************************************
C* MAIN INITIATES DATA READING AND DOES PRELIMINARY ERROR CHECKING
C* ON THE DATA. IT INITIALIZES THE SOLUTION ROUTINE FOR THE PROBLEM
C* PRESENTED AND CARRIES THE OPERATION THROUGH SEVERAL CYCLES OF
C* SOLUTION.
C******************************************************************************
C
C******************************************************************************
C* ALL UNITS ARE IN POUNDS, INCHES OR DEGREES.
C******************************************************************************
C
C******************************************************************************
C* DATA FORMATS ARE 14 FOR INTEGER
C* F10 FOR REAL
C* E15 FOR EXPONENTIAL
C******************************************************************************
C
C******************************************************************************
DIMENSION INC(800,4),NEI(800,5),STIFF(8,8),A(2,1600),CON(2,10)
1 ,IL(2),IH(2),IND(4),TMOE( 800),ICAN(2,75),FORXY(2,75),TL( 800),
2TM( 800),ITSB( 800),PHIS( 800),POS( 800),TSTIFF(800),IRO(50),
3TSTIF1(800),JTAN(75),TMOES(800)
DIMENSION EE(1600),TRI(42000),X(1600),MB(1600)
COMMON C1,C2,C3,STIFF,A,SQ2,CON,TMOE,PO,TLM,FORXY,PLEVEL,FRLEV,
1TL,TM,SMAX,SPLEV,WTBAL,P4X,P4Y,PLX,PLY,PHI,PHID,PHIS,POS,ER,TMOI,
2TSTIFF,TLG,TLF,RMOI,RF,TSTIF1,SPC,SPT,TMOES
COMMON IZERO,ICASE,IDIR,IREEB,INC,NEI,IND,NELS,NELS2
1 ,NOEQ,INEQ,INED2,N12,IL,IH,IEQ,IKW,IKW1,ITOE,IENL,ICAN,ICANI
2 ,NODE1,ICASE3,IRMAT,ITSB,LSYM1,LSYM2,IRO,IRMAT5,IRMAT6,JTAN,KTOS
C
REAL*8 EE,TRI,X
CALL EXCORZ(ICR)
WRITE (6,1001) ICR
1001 FORMAT (1HO,' K BYTES OF UNUSED CORE = ', I10)
LIMIT=47000
SQ2=2.0**0.5*0.5
WRITE (6,67)
47 FORMAT (1H0,24H NUMBER OF SOLUTION SETS )
READ (5,1) NOSET
WRITE (6,61) NOSET
61 FORMAT (I12)
DO 15 IJK1=1,NOSET
PLEVEL=0
WRITE (6,648)
48 FORMAT (1H0,37H MODEL SIZE, NODES DEEP BY NODES WIDE )
READ (5,1) ISHLN, IOLLN
WRITE (6,662) ISHLN, IOLLN
62 FORMAT (I120,114)
WRITE (6,649)
49 FORMAT (1H0,40H TOP NODES ADJACENT TO PLANE OF SYMMETRY )
READ (5,1) LSYM1, LSYM2
WRITE (6,663) LSYM1, LSYM2
63 FORMAT (2110)
IF (LSYM1 .NE. LSYM2) GO TO 41
IF (LSYM1 .EQ. 0) GO TO 41
WRITE (6,642)
42 FORMAT (1H0,29H LSYM1 = LSYM2 ... PROGRAM STOP )
CALL EXIT
C ISHLN = NO. OF NODES VERTICALLY (SHORT)
1 FORMAT (514)
41 NELL=IOLLN-1
NELS=ISHLN-1
C NELL NO. OF NODES (STRESS PTS. LONG WAYS (HORI.
ITE= NELL*NELS
IENO=0
C READ (5,1) IFEY
C WRITE (6,1) IFEY
IFEY=0
IFIY=-IFEY*NELS
WRITE (6,550)
50 FORMAT (1H0,23H VARIABLES, ICASE,IZERO )
READ (5,1) ICASE, IZERO
WRITE (6,664) ICASE, IZERO
64 FORMAT (1H15,17)
WRITE (6,651)
51 FORMAT (1H0,98H RAIL MODULUS RAIL MOMENT OF INERTIA TIE LENGTH
1 TIE LOAD FACTOR TIE SPACING SPRINGS PER TIE )
READ (5,1) ER,TMOI, TLG,TLF,SPC,SPT
36 FORMAT (1H15,2,5F10,3)
WRITE (6,665) ER,TMOI, TLG,TLF,SPC,SPT
65 FORMAT (1F12,2,18,2,F18,1,3F15,2)
RF=2.0*TLF/TLG
RMOI=TMOI*RF
WRITE (6,542)
52 FORMAT (1H0,55H GRID SPACING NUMBER OF LOAD LEVELS WEIGHT OF BAL
1 LAST )
READ (5,11) TLAM,PLEV,WTBAL
WRITE (6,668) TLAM,PLEV,WTBAL
56 FORMAT (1H10,6,18,2,F22,2)
FREL=0.0001
WTBAL=WTAL/1728.0
MAXN=IOLLN*(15MLN-1)+2*IFEY
IF (LSYM1 .NE. 0) MAXN=MAXN-IFEY
NDEQ=MAXN*2
DO 7 I=1,MAXN
7  NEI(I,1)=0
DO 37 I=1,MAXN
TSTIF(I)=0.0
37  TSTIFF(I)=0.0
II=0
WRITE (6,53)
53  FORMAT (1HO,25H TIE NODE AND TIE MODULUS )
39  READ (5,35) I,TSTIF2
WRITE (6,67) I,TSTIF2
67  FORMAT (17,E17.4)
35  FORMAT (14,E15.2)
II=II+1
JTAN(II)=I
IF (I.EQ.0) GO TO 38
TSTIF1(I)=TSTIF2
TSTIFF(I)=TSTIFF2
GO TO 39
38  DO 40 I=1,70
40  TSTIFF(I)=TSTIFF(I)*0.5
KTOE=II
PI=4.0*ATAN(1.0)
WRITE (6,54)
54  FORMAT (1HO,59H BALLAST MODULUS POISSON RATIO ANGLE OF INTERNAL
1FRICTION )
READ (5,11) E,PO,PHI
WRITE (6,68) E,PO,PHI
68  FORMAT (13,F15.3,F23.2)
IF (PO.GE.0.50) GO TO 74
PHI=PHI+2.0*PI/360.0
SPEE=E/40000.0
DO 17 I=1,ITOE
17  POS(I)=PO
PHIS(I)=PHI
17  TMDE(I)=E
WRITE (6,55)
55  FORMAT (1HO, ' INITIAL NODE ELASTIC MODULUS POISSON RATIO AND',
1' ANGLE FOR RANDOM LAYERS' )
23  READ (5,22) J,E,PO,PHI
WRITE (6,69) J,E,PO,PHI
69  FORMAT (110,F18.0,F15.3,F14.2)
IF (PO.GE.0.50) GO TO 74
PHI=PHI+2.0*PI/360.0
IF (J.EQ.0) GO TO 19
DO 18 I=J,ITOE,NELS
18  POS(I)=PO
PHIS(I)=PHI
18  TMDE(I)=E
GO TO 23
19  WRITE (6,56)
56  FORMAT (1HO,64H NODE ELASTIC MODULUS POISSON RATIO AND ANGLE FOR
1 RANDOM NODES )
READ (5,22) J,E,PO,PHI
WRITE (6,70) J,E,PO,PHI
70  FORMAT (14,F16.0,F15.3,F13.2)
IF (PO.GE.0.50) GO TO 74
IF (J.EQ.0) GO TO 20
  
POS(J)=PO
PHI(S,J)=PHI*2.0*PI/360.0
TMOE(J)=E
GO TO 19
21 FORMAT(1X,5I5,F10.0)
22 FORMAT(14,F10.0,2F10.3)
20 DO 2 I=1,NELS
  DO 3 J=1,NELS
  IEN=IEND+1
  J=(I-1)*NELS+JJ+IFEY
  I=J+1
  K=J*NELS
  L=K+1
  IF (J,L.T.0) J=0
  IF (I,L.T.0) I=0
  IF (IFEY.EQ.0) GO TO 16
  IF (I,.NE.NELL,.OR,.LSYM1,.NE.0) GO TO 16
  K=0
  L=0
16 IF (JJ,.NE.NELS) GO TO 4
  I=0
  L=0
  INC(IEND,1)=I
  INC(IEND,2)=J
  INC(IEND,3)=K
  INC(IEND,4)=L
3 CONTINUE
2 CONTINUE
  DO 5 I=1,ITOE
  TMOES(I)=TMOE(I)
  ITSBI=0
  DO 6 J=1,4
    IJK=INC(I,J)
    IF (IJK,.EQ.0) GO TO 6
    NEI(IJK,1)=NEI(IJK,1)+1
    K=NEI(IJK,1)+1
  NEI(IJK,K)=I
5 CONTINUE
4 CONTINUE
C WRITEOUT OF INCIDENCE TABLES
C  DO 8 I=1,ITOE
C  8 WRITE (6,21) I,(INC(I,J),J=1,4),TMOE(I)
C  9 FORMAT (1X,6I5)
C  DO 10 I=1,MAXN
C  10 I=NEI(I,1)+1
C  10 FORMAT (6,9) I,(NEI(I,J),J=1,I)
  N=NODEQ
11 FORMAT (F10.0,2F10.3)
  DO 12 I=1,8
    DO 12 J=1,8
  12 STIFF(I,J)=0.
  I=0
C WRITE (6,57)
57 FORMAT (1HO,49H ICASE, NODE LOADED LOADS IN X AND Y DIRECTIONS )
24 READ (5,25) ICASE3,NODE1,PLX,PLY
C WRITE (6,71) ICASE3,NODE1,PLX,PLY
FORMAT (I5,112,F15.2,F14.2)
I=I+1
ICAN (1,I)=ICASE3
ICAN (2,I)=NODE1
IF (I.EQ.1) GO TO 45
IF (ICAN(2,I-1).LE.ICAN(2,I)) GO TO 45
IF (NODE1.I.EQ.0) GO TO 45
WRITE (6,46)
FORMAT (1HO,48H LOADS OUT OF ORDER, SEQUENTIAL NODE NOS. REQD )
GO TO 75
FORXY(1,I)=-PLX*RF
FORXY(2,I)=-PLY*RF
IF (ICASE3.NE.0) GO TO 24
I=0
WRITE (6,58)
FORMAT (1HO, ' NODE NUMBER AND NODE FIXITY CONDITION 1= FIXED',
1' IN X 2= FIXED IN Y DIRECTION' )
READ(5,25) NODE1,IDIR1
WRITE (6,72) NODE1,IDIR1
FORMAT (I7,I18)
I=I+1
IRO(I)=(NODE1-1)#2+IDIR1
IF (I.EQ.1) GO TO 59
IF (IRO(I-1).LE.IRO(I)) GO TO 59
IF (NODE1.I.EQ.0) GO TO 59
WRITE (6,60)
FORMAT (1HO,44H SEQUENTIAL ORDER OF FIXED NODES IS REQUIRED )
GO TO 75
IF (NODE1.NE.0) GO TO 43
IRO(I)=15000
DO 27 I=1,ITOE
TM(I)=2.0
27 TL(I)=2.0
S MAX=0.0
PLEVEL=1.0
WRITE (6,30) PLEVEL
IKW1=0
IKW=0
N12=-1
IEQ=2
ICOUNT=1
ICANI=0
IZER0=1
IRMAT5=1
IRMAT6=IRO(I)
CALL SOLVE(EE,TRI,X,MB,N,LIMIT)
C PLEVEL=(0.01*FRLEV/S MAX)*1.001
C RAT=PLEVEL/0.01
PLEVEL=1.0
RAT=1.0
IF (PLEVEL.LE.1.0) GO TO 32
PLEVEL=1.0
RAT=100.0
32 DO 31 I=1,NDEQ
31 X(I)=X(I)*RAT
WRITE (6,30) PLEVEL
C CALL OUTPGE (X,N,N,1,1,1)
PLEV1=(1.0-PLEVEL)/PLEVI
PLEVI=0.25
PLEVEL=0.75
C DO 34 I=1,ITU
C34 TM(I)=2.0
C 26 PLEVEL=PLEVI+PLEVEL
   ICOUNT=ICOUNT+1
   WRITE (6,30) PLEVEL
30 FORMAT (/1X,7HLEVEL=,F10.3)
   IKW=0
   IKW=0
   N12=-1
   IEQ=2
   ICANI=0
   IRMATS=1
   IRMATS6=IRO(1)
   IF (ICOUNT.EQ.5) IZERO=0
   CALL SOLVE
      CALL SOLVE(EE,TRI,X,MB,N,LIMIT)
   DO 28 I=1,ITU
28 WRITE (6,29) I,TMDE(I),TL(I),TM(I)
29 FORMAT (/X,15,F10.0,2F10.5)
   IF (ICOUNT.EQ.5) GO TO 34
33 IF (PLEVEL.LT.0.99) GO TO 26
34 IZERO=0
   CALL OUTPGE (X,N,N,1,1,1)
15 CONTINUE
   GO TO 76
74 WRITE (6,73)
73 FORMAT (1HO,'POISSON RATIO EXCESSIVE' )
75 STOP
76 WRITE (6,77)
77 FORMAT (1HO,'EXECUTION COMPLETED' )
STOP
END
SUBROUTINE EQSU

C******************************************************************************
C* EQSU PREPARES THE EQUATIONS FOR INROW TO BE PASSED TO SOLVE.             *
C******************************************************************************

DIMENSION INC(800,4),NEI(800,5),STIFF(8,8),A(2,1600),CON(2,10)
1,IL(2),IH(2),IND(4),TMOE(800),ICAN(2,75),FORXY(2,75),TL(800),
2,TM(800),ITSB(800),PHIS(800),POS(800),TSTIFF(800),IRO(50),
3,TSTIFF1(800),JTAN(75),TMOES(800)
COMMON C1,C2,C3,STIFF,A,SQ2,CON,TMOE,PO,TLAM,FORXY,PLEVEL,FRLEV,
1,TL,TM,SMAX,SLEVE,WTBAL,P4X,P4Y,PLX,PLY,PHI,PHID,PHIS,POS,ER,TMOI,
2,TSTIFF,TLF,RM0I,RF,TSTIFF1,SPC,SPT,TMOES
COMMON IZERO,ICASE,getID,IRECB,INC,NEI,IND,NELS,NELS2
1,NOEQ,IENO,IEN2,N12,IL,IH,IEQ,IKW,IKW1,I1OE,IELN,ICAN,ICANI
2,NODE1,ICASE3,IRMAT,ITSB,LSYM1,LSYM2,IRO,IRMAT5,IRMAT6,JTAN,KTOS

N12=NI2+2
NREC=NI2
NODE=(NI2-1)/2+1
IB1=NREC-100
IF (IB1.LT.1) IB1=1
IB2=NREC+100
IF (IB2.GT.NOEQ) IB2=NOEQ
DO 1 J=IB1,IB2
DO 1 I=1,2
1 A(I,J)=0.
DO 10 I=1,2
DO 10 J=1,ICASE
10 CON(I,J)=0.
IF (LSYM1.EQ.0) GO TO 20
IF (NODE.LT.LSYM2) GO TO 20
A(1,NI2)=1.
A(2,NI2+1)=1.
IH(1)=NOEQ
IH(2)=NOEQ
NI3=NI2-(LSYM2-LSYM1)*2
A(1,NI3)=1.
A(2,NI3+1)=-1.
IL(1)=NI3
IL(2)=NI3
GO TO 21
20 II=NEI(NODE,1)+1
T1=II-1
DO 3 II=2,T1
IELN=NEI(NODE,12)
IND(1)=INC(I1EN,1)
IND(2)=INC(I1EN,2)
IND(3)=INC(I1EN,3)
IND(4)=INC(I1EN,4)
CALL STSU
DO 5 II=1,4
IJK=II
IF (IND(IJK).EQ.NODE) GO TO 6
C ADD ON LOAD VECTOR
C
CON(2,1)=-T1*0.25*WTBAL*TLAM*TLAM
II=0
NREC1=NREC+1
DO 63 I=NREC,NREC1
II=II+1
IF (IKW.NE.0) GO TO 63
IF (IKW1.NE.0) GO TO 66
ICANI=ICANI+1
ICASE3=ICAN(1,ICANI)
NODE1=ICAN(2,ICANI)
PLX=FORXY(1,ICANI)*PLEVEL
PLY=FORXY(2,ICANI)*PLEVEL
18 FORMAT (214,2F10.0)
9 FORMAT (314)
IF (ICASE3.NE.0) GO TO 78
IKW=1
GO TO 63
78 IKW1=1
P4X=PLX
P4Y=PLY
IRMAT=(NODE1-1)*2+1
66 IF (I-IRMAT) 63,70,71
70 CON(1,ICASE3)=-P4X
CON(2,ICASE3)=-P4Y+CON(2,1)
IKW1=0
GO TO 63
71 IENO=6
IENO2=ICASE3
CALL ERRR
63 CONTINUE
IL(1)=(NODE-NELS-2)*2+1
IL(2)=IL(1)
IH(1)=(NODE+NELS)*2+2
IH(2)=IH(1)
IF (IL(1)) 17,17,11
17 IL(1)=1
IL(2)=1
11 IF (IH(1).LE.NOEQ) GO TO 12
IH(1)=NOEQ
IH(2)=NOEQ
12 IF (IRMAT6.GT.N12+1) GO TO 21
IF (IRMAT6-N12) 24,24,23
IL(1)=N12
IH(1)=N12+1
DO 22 J=1,N12,NOEQ
22 A(1,J)=0.
   CON(1,1)=0.
   A(1,N12)=1.
   IRMAT5=IRMAT5+1
   IRMAT6=IRD(IRMAT5)
   IF (IRMAT6.GT.N12+1) GO TO 21
IL(2)=N12
IH(2)=N12+1
DO 25 J=1,NOEQ
25 A(2,J)=0.
   A(2,IRMAT6)=1.
   CON(2,1)=0.
   IRMAT5=IRMAT5+1
   IRMAT6=IRD(IRMAT5)
   GO TO 21
21 DO 13 I=1,2
   I1=IL(I)
   I2=IH(I)
   K=0
   IF (A(I,NREC-I+1).GE.0) GO TO 26
   WRITE (6,27) NREC
   27 FORMAT (1HO,*'NREC =', 110)
26 DO 14 J=1,I1,I2
   K=K+1
   14 A(I,K)=A(I,J)
13 CONTINUE
   IF (PLEVEL.LT.0.5) GO TO 2
   DO 19 I=1,2
   NOEL=IH(I)-IL(I)+1
   N13=N12+I-1
   IF (PLEVEL.LT.0.51) GO TO 19
   WRITEOUT OF EQUATIONS
   WRITE (6,15) N13,IL(I),IH(I),(A(I,J),J=1,NOEL),
   CONTINUE
   WRITE (6,16)
15 CONTINUE
2 CONTINUE
16 FORMAT (1X,3I4,10E10.3/10(13X,10E10.3/))
17 FORMAT (1HO)
RETURN
END
SUBROUTINE ERRR

C

C****************************************************************************
C* ERRR IS A REPOSITORY FOR VARIOUS ERROR MESSAGES AND CHECK PROCEDURES. *
C****************************************************************************

C

DIMENSION INC(800,4),NEI(800,5),STIFF(8,8),A(2,1600),CON(2,10)
1,IL(2),IH(2),IND(4),TMOE(800),ICAN(2,75),FORXY(2,75),TL(800),
2TM(800),ITSB(800),PHIS(800),POS(800),STIFF(800),IRO(50),
3STIFF1(800),JTAN(75),TMODES(800)
COMMON C1,C2,C3,STIFF,A,SQ2,CON,TMOE,PO,TLAM,FORXY,PLEVEL,FRLEV,
1TL,TM,SMAX,SPLEV,MTBAL,P4X,P4Y,PLY,PHI,PHID,PHIS,POS,ER,TMOI,
2STIFF,TLG,TLF,RMOI,RF,TSTIF1,SPC,SPT,TMODE
COMMON IZERO,ICASE,IDIR,IRECB,INC,NEI,IND,NELS,NELS2
1,NODEQ,INEO,IENT,NI2,IL,IM,IEQ,IKW,IKW1,ITOE,IELN,ICAN,ICANI
2,NODEL,ICASE3,IRMAT,ITSB,LSYM1,LSYM2,IRO,IRMAT5,IRMAT6,JTAN,KTOS
WRITE (6,1) IENO,IENT
1 FORMAT (IX,5HERRO,15,10X,15)
RETURN
END
SUBROUTINE STSU

C

C***************************************************************************
C
C   STSU OBTAINS THE ELEMENT STIFFNESS MATRICES FOR ELEMENTS REMAINING
C   PURELY ELASTIC AND FOR THOSE EXPERIENCING TENSILE FAILURE.
C
C***************************************************************************
C
DIMENSION INC(800,4),NEI(800,5),STIFF(8,8),A(2,1600),CON(2,10)
1,IL(2),IH(2),IND(4),TMOE(800),ICAN(2,75),FORXY(2,75),TL(800),
2,TS(800),ITSB(800),PHIS(800),POS(800),TSTIF(800),IRO(50),
3,TSTIF1(800),JTAN(75),TMOES(800)

COMMON C1,C2,C3,STIFF,A,SQ2,CON,TMOE,PO,TLAM,FORXY,LEVEL,FRLEV,
1,TL,TM,SMAX,SLEV,WTL,PA,PL,Y,PLY,PHI,PHID,PHIS,POS,ER,TMOE,
2,TSTIF,TLG,TLF,RMOI,RF,TSTIF1,SPC,SPT,TMOES

COMMON IZERO,ICASE,CT,ITXC,INC,NEI,IND,NELS,NELS2
1,NOEQ,INEO,INEO2,NL2,IL,HI,IEQ,IKW,IKW1,ITO,IELN,ICAN,ICANI
2,NODE1,ICASE3,IRMAT,ITSB,LSYM1,LSYM2,IRO,IRMAT5,IRMAT6,JTAN,KTOS

PO=POS(IELN)
PHI=PHIS(IELN)
TDL=TL(IELN)
TDM=TM(IELN)

TLM=TDL*TDM

IF (((IELN-1)/NELS)*NELS-IELN+1) .EQ. 0) GO TO 9
C1=TMOE(IELN)/(((1.+PO)*(1.-2.*PO))/(1.-PO))
C2=TMOE(IELN)/(((1.+PO)*(1.-2.*PO))*(PO))
CT=TMOE(IELN)/(((1.0+PO)*(1.0-2.0*PO))

T1=(1.0-SIN(PHI))/(1.0+SIN(PHI))

IF (TDL(IELN)*EQ.2.0) GO TO 2
IF (ITSB(IELN)*EQ.2) GO TO 7

CFL=CFL/(6.0)
F0=CF*TDL**4
F1=CF*TLM*TDL*TDL
F2=CF*TLM*TLM
F3=CF*TLM*TDM*TDM
F4=CF*TDM**4

STIFF(1,1)= 2.0*F0+3.0*F1+2.0*F2
STIFF(2,2)= 2.0*F2+3.0*F3+2.0*F4
STIFF(3,3)= 2.0*F0-3.0*F1+2.0*F2
STIFF(4,4)= 2.0*F2-3.0*F3+2.0*F4
STIFF(5,5)= STIFF(1,1)
STIFF(6,6)= STIFF(2,2)
STIFF(7,7)= STIFF(3,3)
STIFF(8,8)= STIFF(4,4)
STIFF(1,2)= 2.0*F1+3.0*F2+2.0*F3
STIFF(1,3)= F0-2.0*F2
STIFF(1,4)= F1-2.0*F3
STIFF(1,5)= F0-3.0*F1-F2
STIFF(1,6)= F1-3.0*F2-F3
STIFF(1,7)= -2.0*F0+F2
STIFF(1,8)= -2.0*F1+F3
STIFF(2,3)= STIFF(1,4)
STIFF(2,4)= F2-2.0*F4
STIFF(2,5)= STIFF(1,6)
STIFF(2,6)=-F2-3.0*F3-F4
STIFF(2,7)= STIFF(1,8)
STIFF(2,8)=-2.0*F2+F4
STIFF(3,4)= 2.0*F1-3.0*F2+2.0*F3
STIFF(3,5) = STIFF(1,7)
STIFF(3,6) = STIFF(1,8)
STIFF(3,7)=-F0+3.0*F1-F2
STIFF(3,8)=-F1+3.0*F2-F3
STIFF(4,5) = STIFF(1,8)
STIFF(4,6) = STIFF(2,8)
STIFF(4,7) = STIFF(3,8)
STIFF(4,8) = F2+3.0*F3-F4
STIFF(5,6) = STIFF(1,2)
STIFF(5,7) = STIFF(1,3)
STIFF(5,8) = STIFF(1,4)
STIFF(6,7) = STIFF(2,3)
STIFF(6,8) = STIFF(2,4)
STIFF(7,8) = STIFF(3,4)
GO TO 6

7 CALL STSU2
GO TO 1

9 CKON=ER*RMO1/(TLAM**3)
TK1=STSTIFF(IIELN)
TK2=STSTIFF(IIELN+NELS)
DO 8 I=1,8
  DO 8 J=1,8
STIFF(I,J)=0.0
CONTINUE

8 STIFF(2,2)=TK1
STIFF(3,3)=4.0*CKON*TLAM*TLAM
STIFF(4,4)=12.0*CKON+TK1
STIFF(5,5)=STIFF(3,3)
STIFF(6,6)=12.0*CKON+TK2
STIFF(8,8)=TK2
STIFF(2,4)=-TK1
STIFF(3,4)=6.0*CKON*TLAM
STIFF(3,5)=2.0*CKON*TLAM*TLAM
STIFF(3,6)=STIFF(3,4)
STIFF(4,5) = STIFF(3,6)
STIFF(4,6) =-12.0*CKON
STIFF(5,6)=STIFF(3,6)
STIFF(6,8)=-TK2
GO TO 6

2 STIFF(1,1)=CT*(3.0-4.0*PO)/6.0
DO 3 I=1,7
  K=I+1
3 STIFF(K,K)=STIFF(I,I)
T1=CT/8.0
T2=CT*PO/6.0
T3=CT*(1.0-4.0*PO)/8.0
T4=CT*(-3.0+4.0*PO)/12.0
T5=-T1
T6=CT*(-3.0+2.0*PO)/12.0
T7=CT*(-1.0+4.0*PO)/8.0
STIFF(1,2)=T1
STIFF(1,3)=T2
STIFF(1,4)=T3
STIFF(1,5)=T4
STIFF(1,6)=T5
STIFF(1,7)=T6
STIFF(1,8)=T7
STIFF(2,3)=T7
STIFF(2,4)=T6
STIFF(2,5)=T5
STIFF(2,6)=T4
STIFF(2,7)=T3
STIFF(2,8)=T2
STIFF(3,4)=T5
STIFF(3,5)=T6
STIFF(3,6)=T3
STIFF(3,7)=T4
STIFF(3,8)=T1
STIFF(4,5)=T7
STIFF(4,6)=T2
STIFF(4,7)=T1
STIFF(4,8)=T4
STIFF(5,6)=T1
STIFF(5,7)=T2
STIFF(5,8)=T3
STIFF(6,7)=T7
STIFF(6,8)=T6
STIFF(7,8)=T5
6     JI=2
DO 4 I=1,7
   DO 5 J=J1,8
 5     STIFF(J,I)=STIFF(I,J)
4     JI=JI+1
1     RETURN
END
SUBROUTINE STSU2

* STSU2 OBTAINS THE ELEMENT STIFFNESS MATRICES FOR THOSE ELEMENTS WHICH EXPERIENCE SHEAR FAILURE.

* *

DIMENSION INC(800,4),NEI(800,5),STIFF(8,8),A(2,1600),CON(2,10)
1,IL(2),IH(2),IND(4),TMOE( 800),ICAN(2,75),FORXY(2,75),TL( 800),
2IM( 800),ITSB( 800),PHIS( 800),POS( 800),STIFF(800),IR0(150),
3STIFF(800),JTNAN(150),TMOE(800)

COMMON C1,C2,C3,STIFF,A1,SQ2,CON,TMOE,PO,TLAM,FRXY,PLEVEL,FRLEV,
1TL,TMSM,SMAX,SPLEV,WSBAL,P4X,P4Y,PLX,PLY,PHI,PHID,PHIS,POS,ER,TMOI,
2STIFF,LG,TLF,RF,STIFIL,SPC,SPT,TMOES

COMMON IZERO,ICASE,DIR,IRECB,INC,NEI,IND,NELS,NELS2
1,NOE0,IE00,IE002,N12,IL,ILH,IE0,IKW,IKW1,ITOE,IELN,ICAN,ICANI
2,NODE1,ICASE3,IRMAT,ITSB,LSYM1,LSYM2,IR0,IRMAT5,IRMAT6,JTN,KTOS

PO=POS(IELN)
PHI=PHIS(IELN)
TDL=TL(IELN)
TDN=TM(IELN)

TDL=TDL*TOL

C1=TMOE(IELN)/((1.0+PO)*(1.0-2.0*PO)) *(1.0-PO)
C2=TMOE(IELN)/((1.0+PO)*(1.0-2.0*PO)) *(PO)
CT=TMOE(IELN)/((1.0+PO)*(1.0-2.0*PO))

T=(1.0-SIN(PHI))/(1.0+SIN(PHI))

C3=T*C1
C4=T*C2
C5=(2*0.25)*C1
C6=(2*0.25)*C3
C7=(2*0.25)*C4
F0=TDL**4
F1=TDL**4
F2=TDL**4
F3=TDL**4
F4=TDL**4
F5=TDL**4
F6=TDL**4
F7=TDL**4
F8=TDL**4
F9=TDL**4
F10=TDL**4
F11=TDL**4
F12=TDL**4
F13=TDL**4
F14=TDL**4

STIFF(1,1)=C1*(F02+F13+F22)+C2*(F3-F1)+C3*(F3-F1)+C4*(F42+F33+F22)
STIFF(2,2)=C1*(F42+F33+F22)+C2*(F1-F3)+C3*(F1-F3)+C4*(F42+F33+F22)
STIFF(3,3)=C1*(F02+F13+F22)+C2*(F1-F3)+C3*(F1-F3)+C4*(F42+F33+F22)
STIFF(4,4)=C1*(F42+F33+F22)+C2*(F3-F1)+C3*(F3-F1)+C4*(F42+F33+F22)
STIFF(5,5)=STIFF(1,1)
STIFF(6,6)=STIFF(2,2)
STIFF(7,7)=STIFF(3,3)
STIFF(8,8)=STIFF(4,4)
STIFF(8,5) = Z1*(F1-F32) + Z2*(-F4+F32-F2) + Z3*(-F0-F12-F2) + Z4*(F12-F3)
STIFF(8,6) = Z1*(F2-F42) + Z2*(F1-F22+F3) + Z3*(-F1-F22-F3) + Z4*(F2-F02)
STIFF(8,7) = Z1*(F12-F23+F32) + Z2*(F2-F4) + Z3*(F2-F0) + Z4*(-F12-F23-F32)
RETURN
END
SUBROUTINE SOLVE (AA,B,C,MB,N,LIMIT)

C
C******************************************************************************
C*          SOLVE FINDS SOLUTIONS FOR THE SIMULTANEOUS EQUATIONS SET UP BY    *
C*          EQSU AND OBTAINED FROM INROW BY GAUSS ELIMINATION TECHNIQUE ON A   *
C*          GENERAL BANDED COEFFICIENT MATRIX.                                *
C******************************************************************************
C
DIMENSION INC(800,4),NEI(800,5),STIFF(8,8),A(2,1600),CON(2,10)
1 ,IL(2),IH(2),IND(4),TMOE( 800),ICAN(2,75),FORXY(2,75),TL( 800),
2TM( 800),ITSB( 800),PHIS( 800),POS( 800),TSTIFF(800),IRO(50),
3TSTIF1(800),JTAN(75),TMOES(800)
DIMENSION AA(1),BB(1),CC(1),MB(1),CONV(1),A1(1600)
COMMON C1,C2,C3,STIFF,A,SQ2,CON,TMOE,PO,TLAM,FORXY,PLEVEL,FRLEV,
1TL,TM,SMAX,SPLVE,TWVAL,P4X,P4Y,PXL,PLY,PHI,PHID,PHIS,POS,ER,TMODI,
2TSTIFF,TLG,TL,RMDI,RF,TSTIF1,SPC,SPT,TMOES
COMMON IZERO,ICASE,DIRC,IRECB,INC,NEI,IND,NELS,NELS2
1,NOEQ,INNO,INNO2,N12,IL,IH,IEQ,IKW,IKM1,ITOE,IELN,ICAN,ICAN
2,NODE1,ICASE3,IRMAT,ITSB,LSYM1,LSYM2,IRO,IRMAT5,IRMAT6,JTAN,KTOS
REAL*8 AA,B,C,CON1
MB(1)=1
DO 20 I=1,NOEQ
  AA(I)=0.0
  DO 1 I=1,N
    CALL INROW(II,KLOW,KHIGH,CONV,A1,NOC)
    KSUM=0
    DO 16 KSUM1=KLOW,KHIGH
      KSUM=KSUM+1
      IF (ABS(A1(KSUM)) .LE.1.0E-10) A1(KSUM)=0.0
    AA(KSUM)=A1(KSUM)
    CON1=-CON1
    KLOW=KLOW
    KHIGH=KHIGH+1-KLOW
    KU=I-1
    IF (KU) 4,4,3
    3 K=0
    IF (KU-KL) 4+2,2
    2 DO 5 K=KLOW,KU
      K=K+1
      LL=MB(K)
      LU=MB(K+1)-1
      L1=K1
      DO 6 L=LL,LU
        L1=L1+1
        AA(L1)=AA(L1)+B(L)*AA(K1)
      IF (L1-KH) 6,6,14
    4 IF (AA(L1)) 15,6,15
    15 KH=L1
    6 CONTINUE
    5 CON1=CON1+AA(K1)*C(K)
    4 K1=I+1-KLOW
    KLOW=KLOW+1
L=MB(I)-1
IF (AA(K1)) 10, 11, 10
11
ERROR=I
WRITE (6,100) ERROR
100 FORMAT (1HO,11HEQUATION NO ,F6.0,26HHAS A ZERO ON THE DIAGONAL )
   AA(K1)=SLEV
   NNN=(I+1)/2
   NNN1=I-(NNN-1)*2
   WRITE (6,102) SLEV, NNN, NNN1
102 FORMAT (1HO,F10.5,28HLB. SPRING ATTACHED TO NODE ,I5,8H IN THE
   1,I5,11H DIRECTION )
10 DO 7 K=KL,KH
   L=L+1
   7 B(L)=-AA(K)/AA(K1)
   MB(I+1)=L+1
   C(J1)=-CON1/AA(K1)
   IF (L-LIMIT) 1, 13, 13
13 ERROR=I
WRITE (6,101) ERROR
101 FORMAT (1H1,16HARRAY B IS FULL ,F8.0,27HINCREASE THE DIMENSION OF
   1 B )
   CALL EXIT
1 CONTINUE
ERROR=0.
N2=N-1
DO 8 I=1,N2
   JA=N-I
   JL=MB(JA)
   JU=MB(JA+1)-1
   J2=JA
   DO 8 J=JL,JU
   J2=J2+1
8 C(J2)=C(JA)+B(J)*C(J2)
   NSTART=1
   WRITE (6,103) L
103 FORMAT (1HO,97HLIMIT =', I10).
   CALL OUTPG(E(C*N,N,NSTART,NSTART,NSTART))
   RETURN
END
SUBROUTINE INROW(I1, I1I, I1H, CON1, A1, NOCI)

C
C***********************************************************************
C*
C* INROW PASSES THE EQUATIONS OBTAINED FROM EQSU TO SOLVE.
C*
C***********************************************************************

C

DIMENSION INC(800,4), NEI(800,5), STIFF(8,8), A(2,1600), CON(2,10)
1, I1L(2), I1H(2), IND(4), TMOE(800), ICAN(2,75), FORXY(2,75), TL(800),
2, TM(800), ITS(800), PHI(800), POS(800), TSTIFF(800), IRO(50),
3, TSTIFF1(800), JTA(75), TM0ES(800)

DIMENSION A1(1600), CON1(10)

COMMON C1, C2, C3, STIFF, A, SQ2, CON, TMOE, PO, TLAM, FORXY, PLEVEL, FRLEV,
1, TL, TM, SMAX, SLEVEL, WTBAL, P4X, P4Y, PLX, PLY, PHI, PHID, PHIS, POS, ER, TMOI,
2, TSTIFF, TLF, TEMO, RF, TSTIFF1, SP, SPT, TM0E

COMMON I0E, ICASE, IDIR, IREC0, INC, NEI, IND, NELS, NELS2
1, NOEQ, IOEQ, IOEQ2, N12, I1H, IEQ, IKW, IKW1, ITOE, IELN, ICAN, ICANI
2, NOEQ, ICASE3, IRMAT, ITS, LSYM1, LSYM2, IRD, IRMAT5, IRMAT6, JTA, KTOS

IF (IEQ-1) 20, 21, 20

21 IEQ=2
22 IEQ=1
GO TO 22

20 IEQ=1

CALL EQSU

NOE=I1H(IEQ)-I1L(IEQ)+1
DO 1 I=1, NOE
1 A1(I)=A(IEQ, I)

CON1(I)=CON(IEQ, I)
I1L=I1L(IEQ)
I1H=I1H(IEQ)
NOCI=1
II=N12
RETURN

END
SUBROUTINE OUTPGE(A1, LENG, NOEQ1, NSTART, NOC1, NSOLN)

C*******************************************************************************
C*******************************************************************************
C* OUTPGE COMPUTES STRESSES FOR THE NODE CENTERS ON THE BASIS OF THE         *
C* DISPLACEMENT FUNCTIONS PREPARED BY SOLVE. CHECKING FOR TENSILE           *
C* AND SHEAR FAILURE MODES IS PERFORMED. CRACKED NODES AND THE CRACK        *
C* DIRECTIONS ARE DOCUMENTED. RAIL MOMENTS AND TIE FORCES ARE COMPUTED       *
C* AND PRINTED. ELEMENTS FAILING IN TENSION ARE ASSUMED TO CARRY NO          *
C* LOAD FOR THE CURRENT CYCLE. A COMPRRESSIVE LOADING WILL REINSTATE         *
C* THESE ELEMENTS IN A LATER CYCLE. THE EQUIVALENT TIE SPRINGS ARE          *
C* ASSUMED TO BE CUT IF REQUIRED TO TAKE TENSILE LOADING, THEY ARE          *
C* EFFECTIVE ONLY IN COMPRESSION.                                          *
C*******************************************************************************

C*******************************************************************************
DIMENSION INC(800,4), NEI(800,5), STIFF(8,8), A(2,1600), CON(2,10)
1, IL(2), IH(2), IND(4), TMOD(800), ICAN(2,75), FORXY(2,75), TL(800),
2TM(800), ITSBI(800), PHISI(800), POSI(800), TSTIFF(800), IRO(50),
3TSTIF1(800), JTAN(75), TMOESI(800)
DIMENSION A1(1600), CONI(1)
DIMENSION DISI(8)
COMMON C1,C2,C3,STIFF,A, SQ2, CON, TMODE, PO, TLAM, FORXY, PLEVEL, FRLEV,
1TL, TM, SMAX, SPLAIN, WTBAL, P4X, P4Y, PLX, PLY, PHI, PHID, PHIS, POS, ER, TMOI,
2TSTIFF1, TLG, TLF, RMFI1, RF, TSTIF1, SPC, SPT, TMOES
COMMON IZERO, ICASE, IDIR, IRECB, INC, NEI, IND, NELS, NELS2
1, NOEQ1, IENO, IENO2, NI2, IL, IH, IEQ1, IKW1, ITOE, IELN, ICAN, ICANI
2, NODE1, ICASE3, IRMAT, ITSBI, LSYM1, LSYM2, IRO, IRMAT5, IRMAT6, JTAN, KTOS
C
REAL*8 A1
PI=4.0*ATAN(1.0)
IF (IZERO.EQ.1) GO TO 31
WRITE (6,38)
FORMAT (1H0,43H NODE X DEFLECTION Y DEFLECTION )
DO 2 J=1, LENG, 2
J1=J+2+1
WRITE (6,1) J1, A1(J), A1(J+1)
2 CONTINUE

31 DO 3 I1=1, ITOE
IND(1)=INC(I1,1)
IND(2)=INC(I1,2)
IND(3)=INC(I1,3)
IND(4)=INC(I1,4)
IELN=I1
TMOD(IELN)=TMODSI(IELN)
PO=POSI(IELN)
PHI=PHISI(IELN)
PHID=PHII*180.0/PI
DO 4 I2=1, 4
DIS(I2*2-1)=0.
DIS(I2*2)=0.
IF (IND(I2), EQ, 0) GO TO 4
DIS(I2*2-1)=A1(IND(I2)*2-1)
DIS(I2*2)=A1(IND(I2)*2)
4 CONTINUE
IF (((IELN-1)/NELS)*NELS-IELN+1), EQ, 0) GO TO 35
SX=0.
SY=0.
SXY=0.
IF (T(IENL).EQ.2.0) GO TO 23
TDL=TL(IENL)
TDM=TM(IENL)
TLM=TDL*TDM
C1=TM0E(IENL)/(1.*PO)*(1.-2.*PO)*(1.-PO)
C2=TM0E(IENL)/(1.*PO)*(1.-2.*PO)*(PO)
T=(1.0-SIN(PHI))/(1.0+SIN(PHI))
IF (ITSB(I1).EQ.2) GO TO 28
CF=C1/(2.0*TLM)
F0=CF*TDL**4
F1=CF*TLM*TDL*TDL
F2=CF*TLM*TLM
F3=CF*TLM*TDMSL*TDM
F4=CF*TDM**4
STIFF(1,1)=-F0-F1
STIFF(1,2)=-F2-F1
STIFF(1,3)=-F0+F1
STIFF(1,4)= F2-F1
STIFF(1,5)= F0+F1
STIFF(1,6)= F2+F1
STIFF(1,7)= F0-F1
STIFF(1,8)=-F2+F1
STIFF(2,1)=-F2-F3
STIFF(2,2)=-F4-F3
STIFF(2,3)=-F2+F3
STIFF(2,4) = F4-F3
STIFF(2,5) = F2+F3
STIFF(2,6) = F4+F3
STIFF(2,7) = F2-F3
STIFF(2,8) =-F4+F3
STIFF(3,1)=-F1-F2
STIFF(3,2)=-F3-F2
STIFF(3,3)=-F1+F2
STIFF(3,4)= F3-F2
STIFF(3,5)= F1+F2
STIFF(3,6) = F3+F2
STIFF(3,7) = F1-F2
STIFF(3,8)=-F3+F2
GO TO 24
C3=T*C1
C4=T*C2
CC=0.5/TLM
F0=TDL**4
F1=TLMSL*TDL
F2=TLM*TLM
F3=TLM*TDM*TDM
F4=TDM**4
E1=CC*(C1+F0+C2+F2+C3+F2+C4+F4)
E2=CC*(C1+F2+C2+F0+C3+F4+C4+F2)
E3=CC*(C1+F1-C2+F1+C3+F3-C4+F3)
E4=CC*(C1+F2+C2+F4+C3+F0+C4+F2)
E5=CC*(C1+F4+C2+F2+C3+F2+C4+F0)
E6=CC*(C1+F3-C2+F3+C3+F1-C4+F1)
E7=CC*(C1+F1+C2+F3-C3+F1-C4+F3)
E8 = CC*(C1*F3+C2*F1-C3*F3-C4*F1)
E9 = CC*(C1*F2-C2*F2-C3*F2+C4*F2)
STIFF(1,1) = -E1-E3
STIFF(1,2) = -E2-E3
STIFF(1,3) = -E1+E3
STIFF(1,4) = +E2-E3
STIFF(1,5) = +E1+E3
STIFF(1,6) = +E2+E3
STIFF(1,7) = +E1-E3
STIFF(1,8) = -E2+E3
STIFF(2,1) = -E4-E6
STIFF(2,2) = -E5-E6
STIFF(2,3) = -E4+E6
STIFF(2,4) = +E5-E6
STIFF(2,5) = +E4+E6
STIFF(2,6) = +E5+E6
STIFF(2,7) = +E4+E6
STIFF(2,8) = -E5+E6
STIFF(3,1) = -E7-E9
STIFF(3,2) = -E8-E9
STIFF(3,3) = -E7+E9
STIFF(3,4) = +E8-E9
STIFF(3,5) = +E7+E9
STIFF(3,6) = +E8+E9
STIFF(3,7) = +E7-E9
STIFF(3,8) = -E8+E9
GO TO 24

23 CT = TMOE (IELN) / ((1.0+PO)*(1.0-2.0*PO))/TLAM
T1 = -CT*(1.0-PO)*0.5
T2 = -CT*PO*0.5
T3 = -CT*(1.0-2.0*PO)*0.25
STIFF(1,1) = T1
STIFF(1,2) = T2
STIFF(1,3) = T1
STIFF(1,4) = -T2
STIFF(1,5) = -T1
STIFF(1,6) = -T2
STIFF(1,7) = -T1
STIFF(1,8) = T2
STIFF(2,1) = T2
STIFF(2,2) = T1
STIFF(2,3) = T2
STIFF(2,4) = -T1
STIFF(2,5) = -T2
STIFF(2,6) = -T1
STIFF(2,7) = -T2
STIFF(2,8) = T1
STIFF(3,1) = T3
STIFF(3,2) = T3
STIFF(3,3) = -T3
STIFF(3,4) = T3
STIFF(3,5) = -T3
STIFF(3,6) = -T3
STIFF(3,7) = T3
STIFF(3,8) = -T3
DO 6 I3 = 1, 8
SX = SX + STIFF(1,I3)*DIS(I3)
SY=SY+STIFF(2,13)*DIS(I3)
6 SXY=SXY+STIFF(3,13)*DIS(I3)
S1=(SX+SY)*0.5
S2=0.5*((SX-SY)**2+4.0*SXY*SXY)**0.5
S3=S1+S2
S4=S1-S2
IS3=1
IS4=1
I7=2
IF (INC(IELN,I),EQ.0) I7=3
I6=(INC(IELN,I)-1)/NELS
I6=INC(IELN,I)-16*NELS
I6=I6
FRLEV=((T6-1.0)+0.5)*TLAM*WTBAL
SX=SX-FRLEV
SY=SY-FRLEV
IF (S3.GT.SMAX) SMAX=S3
IF (S4.GT.SMAX) SMAX=S4
IF (IERO.EQ.1) GO TO 32
WRITE (6,1) I1,SX,SY,SXY,S3,S4
32 IF (S3-FRLEV) 10,8,8
8 IS3=0
10 IF (S4-FRLEV) 9,11,11
11 IS4=0
C BOTH DIRECTIONS CRACK
9 IF (IS3+IS4) 13,13,26
12 IF(IS3+IS4-2) 27,26,26
13 TMOE(IELN)=0.0
PLEVEL=0.75
WRITE (6,15) IELN
15 FORMAT (1HO,25HSTRESS POINT WIPEP OUT AT ,I5)
GO TO 3
27 ITSB(I1)=1
14 TI=IS3
T2=IS4
S1=S3*T1+S4*T2
IF (T1(I1).EQ.2.0) GO TO 21
IF (T5(I1).EQ.2) GO TO 21
WRITE (6,72) I1
22 FORMAT (1HO,12HSTRESS POINT ,I5,5X,13HCRAOKE AGAIN )
GO TO 13
21 ROLEV=0.1*PLEVEL
IF (KROLEV.GT.0.001) ROLEV=0.001
IF (S5(I1).EQ.TLEV) 16,16,17
16 TL(I1)=1.0
TM(I1)=0.0
TAN=0.0
GO TO 28
17 TMI=1.0/((SX/(SX-S1)**2)
TII=(1.0-TM1)**0.5
TAN=45.0
TM(11)=1.0
IF (AM5/I1111.LE.0.001) GO TO 25
TMI=1-S1*TL(11)/SX
TAN=ATAN(TM(I1)/TM(II))*ATAN(TM(II)/TM(I1))
25 TAN=TAN+5.0-PHR/2.0
TAN2=TAN-45.0-PHR/2.0
WRITE (6,30) I1,TAN1,TAN2
30  FORMAT (1HO,39HSHEAR CRACK DIRECTIONS IN STRESS POINT ,I5,S5X,3MAR
1E,F10.2,S5X,3HANG,F10.2,S5X,18HCC FROM HORIZONTAL )
WRITE (6,19) I1,TAN
19  FORMAT (1HO,31HCRAK DIRECTION IN STRESS POINT ,I5,S5X,2HIS, F10.2
1S5X,18HCC FROM HORIZONTAL )
ITSB(I1)=2
PLEVEL=0.75
GO TO 3
26  RAD=ABS((S4-S3)*0.5)
OPI=ABS((S4+S3)*0.5*SIN(PHI))
IF (RAD.LT.OPP) GO TO 3
WRITE (6,29)
29  FORMAT (/1X,18HSHEAR CRACK FORMED )
ITSB(I1)=2
GO TO 14
35  TF1=0.0
TM1=0.0
TM2=0.0
TF2=0.0
C IF (IZERO.EQ.1) GO TO 3
CKON=ER*TM01/(TLAM**3)
TK1=STIFF(I1)
TK2=STIFF(I1+NELS)
DO 36 I=1,4
DO 36 J=1,8
STIFF(I,J)=0.0
36  CONTINUE
STIFF(1,2)=TK1
STIFF(1,4)=TK1
STIFF(2,3)=4.0*CKON*TLAM
STIFF(2,4)=6.0*CKON*TLAM
STIFF(2,5)=2.0*CKON*TLAM
STIFF(2,6)=-STIFF(2,4)
STIFF(3,3)= STIFF(2,5)
STIFF(3,4)= STIFF(2,4)
STIFF(3,5)= STIFF(2,3)
STIFF(3,6)= STIFF(2,6)
STIFF(4,6)= TK2
STIFF(4,8)=-TK2
DO 37 II=1,8
TF1=TF1+STIFF(1,II)*DIS(II)
TM1=TM1+STIFF(2,II)*DIS(II)
TM2=TM2+STIFF(3,II)*DIS(II)
TF2=TF2+STIFF(4,II)*DIS(II)
37  CONTINUE
TF1=2.0*TF1
TF2=2.0*TF2
IF (DIS(4).*LE,DIS(2)) STIFF(I1)=STIFF(I1)
IF (TF1.GT.0.0) STIFF(I1)=0.0
IF (TF2.GT.0.0) STIFF(I1+NELS)=0.0
IF (IZERO.EQ.1) GO TO 3
WRITE (6,40)
40  FORMAT(1HO,43H NODE TIE FORCE RAIL MOMENT, 8X,44HT
1IE FORCE RAIL MOMENT (NODE + 1) )
WRITE (6,1) I1,TF1,TM1,TF2,TM2
WRITE (6,39)
FORMAT (1HO, 20H NODE STRESS X, 10X, 9H STRESS Y, 11X, 12H SHEAR S)
1 STRESS, 9X, 26H PRINCIPAL STRESSES X AND Y
3 CONTINUE
1 FORMAT (1X, I5, 6(5X, E14.7))
WRITE (6, 41)
41 FORMAT (1HO, ' NODE AND VERTICAL SURFACE DEFLECTION')
NELS2 = NELS*2
DO 45 KKK = 2, 4, 2
JSB = KKK
DO 42 I = JSB, NOEQ, NELS2
NODE = I/2
WRITE (6, 43) NODE, A(I)
C
WRITE (7, 43) NODE, A(I)
43 FORMAT (1X, I5, 5X, E14.8)
42 CONTINUE
CALL CALMOD(A(I), JSB)
45 CONTINUE
RETURN
END
SUBROUTINE CALMOD (AI,JSB)

C

C***********************************************************************************************************************************************************
C
C* CALMOD COMPUTES THE TRACK MODULUS VALUE BY THE EQUILIBRIUM EQUATION --- SUM LOADS EQUALS SUM DEFLECTIONS TIMES SPACING BETWEEN
C* DEFORMATION MEASUREMENTS TIMES TRACK MODULUS.
C
C***********************************************************************************************************************************************************

DIMENSION INC(800,4),NEI(800,5),STIFF(8,8),A(2,1600),CON(2,10)
1,IL(2),IH(2),IND(4),TMOE( 800),ICAN(2,75),FORXY(2,75),TL( 800),
2,TM( 800),ITSB( 800),PHIS( 800),POS( 800),TSTIFF(800),IRO(50),
3,TSTIF1(800),JTAN(75),TMOES(800)
DIMENSION AI(1600)
COMMON C1,C2,C3,STIFF,A,SQ2,CON,TMOE,PO,TLAM,FORXY,PLEVEL,FRLEV,
1,TL,TM,SMAX,SPLEV,WTBAL,P4X,P4,Y,PLX,PLY,PHI,PHIO,PHIS,POS,ER,TMOI,
2,TSTIFF,TLG,TL,TMOI,RF,TSTIF1,SPC,SPT,TMOES
COMMON IZERO,ICASE,DIR,IRECB,INC,NEI,IND,NELS,NELS2
1,NODEQ,NED12,N12,IL1H1,EQ,IW,IKW,ITOE,IELN,ICAN,ICANI
2,NODE1,ICASE3,IRMAT,ITSB,LSYM1,LSYM2,IRO,IRMAT5,IRMAT6,JTAN,KTOS
NDEC=LSYM1+LSYM2/2
TMULT=2.0
IF (((LSYM1+LSYM2) .NE. 0) GO TO 4
TMULT=1.0
NDEC=10000
4 YSUM=0.0
YSUMSQ=0.0
DO 1 I=JSB,NODEQ,NELS2
II=I
NODE=I/2
IF (NODE.EQ.NODEQ) GO TO 2
IF (NODE.GT.NODEQ) GO TO 1
YSUM=YSUM+AI(I)*TMULT
YSUMSQ=YSUMSQ+AI(I)**2*TMULT
1 CONTINUE
GO TO 3
2 YSUM=YSUM+AI(I)
YSUMSQ=YSUMSQ+AI(I)*AI(I)
3 SFC=0.0
SFCD=0.0
I=0
5 I=I+1
IF (ICAN(2,I).GT.NODEQ) GO TO 6
NODEN=ICAN(2,I)**2
IF (ICAN(2,I).EQ.NODEQ) TMULT=1.0
FCE=FORXY(2,I)/RF*TMULT
SFC=SFC+FCE
SFCD=SFCD+FCE*AI(NODEN)
IF (ICAN(2,I+1).NE.0) GO TO 5
U1=-SFC/(YSUM*TLAM)
U2=-SFCD/(YSUMSQ*TLAM)
WRITE (6,7) U1
7 FORMAT (1HO,'TRACK MODULUS - EQUILIBRIUM METHOD',/1HO,F20.0)
YSUM=0.0
DO 8 I=1,KTOS
TMULT=1.0
IF (JTAN(I) .LE. LSYM1) TMULT=2.0
IF (JTAN(I) .GE. LSYM2) TMULT=0.0
I1=JTAN(I)*2+2
8 YSUM=YSUM+A1(I1)*TMULT
U3=-SFC*SPT/(SPC+YSUM)
WRITE (6,9) U3
9 FORMAT (1HO,* TRACK MODULUS - TIE DEFLECTION /1HO,F20.0)
RETURN
END
/*
//GO.SYSUDUMP DD SYSOUT=A
//GO.SYSIN DD *