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VERTICAL PRESSURE DISTRIBUTION IN THE BALLAST SECTION AND ON THE SUBGRADE BENEATH STATICALLY LOADED TIES

by M. T. Salem and W. W. Hay

Conducted by

Railway Research
Department of Civil Engineering
Engineering Experiment Station
University of Illinois

in cooperation with

The Railway Maintenance Corporation
Association of American Railroads
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University of Illinois
Urbana, Illinois
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ACKNOWLEDGEMENTS

This report concerns a project conducted by M. T. Salem as a doctoral thesis research. The research was sponsored by the Railway Maintenance Corporation of Pittsburgh, Pennsylvania through its President, Mr. Paul S. Settle, Jr. Pressure cells and calibrating equipment were furnished by Mr. G. M. Magee, Director of Engineering Research at the Research Center of the Association of American Railroads in Chicago. Ties, rails, ballast and technical assistance were provided by the Illinois Central Railroad acting through the Chief Engineer, Mr. Allen Sams, Division Engineer, P. A. Cosgrove and Roadmaster Felix Fortino. Their support is noted with appreciation and thanks.

On the part of the University, the work covered in this report was carried out under the general administrative supervision of W. L. Everitt, Dean of the College of Engineering, Ross J. Martin, Director of the Engineering Experiment Station, N. M. Newmark, Head of the Department of Civil Engineering, and W. W. Hay, Professor of Railway Civil Engineering.

INTRODUCTION

A program to obtain more nearly definitive measures of ballast pressure distribution utilizing modern types of earth pressure cells and laboratory simulation of a well-compacted subgrade under the ballast was undertaken in the Civil Engineering Department at the University of Illinois during the period of February 1965 through June 1966. The actual work was carried out by Dr. M. T. Salem, doctoral candidate, under the direction of Dr. W. W. Hay, Professor of Railway Civil Engineering. The study was given financial sponsorship by the Railway Maintenance Corporation of Pittsburgh, Pennsylvania, pressure cells were loaned by the Research Center, Association of American Railroads, Chicago, Illinois and rails, ties, ballast and subgrade materials were donated by the Illinois Central Railroad. The hydraulic loading jacks, 12-channel pressure cell recorder and a working site were provided by the Civil Engineering Department, University of Illinois.

The ballast section in its primary functions of distributing wheel loads from the cross ties, anchoring the track, providing immediate drainage, and contributing resiliance is one of the most important elements in a railroad track structure. A suitable depth of ballast is needed in order to get a uniform pressure distribution on the subgrade under the rail and a unit pressure distribution imposed on the subgrade within the allowable limits of the subgrade bearing capacity. If ballast pressures are not uniformly distributed on the subgrade, some of the ballast particles will be forced into the subgrade leading to instabilities and formation of ballast pockets.

The design of most engineering structures must, of necessity, take into consideration the quality and nature of the support offered by the soil on which the structure is to rest. It follows that the ballast section should be laid on a well-prepared subgrade; weak or unstable natural ground or subgrade can cause successive subsidence or sliding. Usually in the railway field, the uniformity of the support is more important than the actual unit supporting values, although both are of interest.

PREVIOUS WORK

Considerable attention has been given to various features of the ballast section over the years. Schubert's studies in Germany on comparison of pressure distribution by different ballasts in 1887-1890, Brauning's work with gravel as a ballast in 1904, and the full-scale ballast box tests of the Pennsylvania Railroad at Altoona in 1912 are representative of these efforts. The work of the Joint Committee on Stresses in Track has been a landmark for United States railroad engineering.

Dr. A. N. Talbot, Chairman, and his Joint Committee on Stresses in Track carried out in 1919 an extensive study on the pressure distribution in the ballast section. From these studies an equation was developed to determine the pressure at any point below and to the right or left of a particular tie. The ballast section was placed on a 12-inch sand cushion in some tests and directly on a concrete floor in others. The latter represents a special case because most railroad embankments are not that rigid. This factor, coupled with a probable ballast section distortion occasioned by the cumbersome type of pressure cell and recording devices available at that time could have introduced inaccuracies into the results obtained. It was concluded by the Committee that a depth of ballast equal to the tie spacing plus three to four addition inches of ballast depth would be needed to obtain a reasonably uniform pressure under the rail on the subgrade.

PURPOSE AND SCOPE

The purpose of this study was to make a modern determination of vertical pressure distribution through the ballast section to the subgrade beneath statically loaded ties and to determine the depth of ballast needed to attain a reasonably uniform pressure distribution under the rails on the subgrade for any particular tie spacing. An equation, based on theory and test results, was developed by means of which the vertical pressure below and to the right and left of the center line of a tie can be determined. The study concerned itself with wood ties placed on three different types of ballast and loaded with a static load.

TEST PROCEDURES

The vertical pressure distribution in the ballast section and on the subgrade was determined for three types of ballast, namely chat, pit run gravel, and crushed slag. Three wood ties 7 inches by 9 inches by 8.5 feet were used at tie spacings of 19.5, 21, and 22 inches. A static load was applied by a hydraulic jack used with a calibrated dynamometer to determine the load which was applied on one tie and also on three ties in a group, Fig. 1. The load applied on the three ties ranged from 10,000 to 25,000 pounds, and that applied on one tie ranged from 6,666 to 20,000 pounds. The subgrade material used was classified as A-7-6 according to the AASHO classification. The properties of soil are shown in the following table:

Subgrade Soil	Water Content	Liquid Limit	Plastic Limit	Shrinkage Limit	Optimum Moisture Content	Maximum Dry Content	Classification According to AASHO
silty-clay	28.5	45	26	17	19.8	97.9	A-7-6

It was placed in 3-inch lifts and compacted thoroughly to get a 97 percent degree of compaction, as determined by the standard compaction test, on the dry side of optimum. The average dry density was 95.5 pcf. Ballast was placed on the subgrade and tamped in layers of not more than 6 inches. Tests were made for 12, 18, 24, and 30 inches of ballast depth beneath the ties. Pressure cells used were developed by the staff of the A.A.R. Research Center in Chicago. The cells were calibrated under air and sand pressures. Only the calibration under sand pressure was used for the conversion of the test data. The cells were placed on the subgrade, Fig. 2, and in the ballast section after being bedded and covered by 2 1/2 inches of sand of the same size as that used for calibration.

THEORETICAL ANALYSIS

Two equations were developed for this study. These take into consideration both theory and practice by introducing the correction factors C and K into the equations, thereby relating test results to theoretical evaluations.

Because the load is applied on the ties at two points (by two rails), the pressure on the base area of the tie is by no means uniform. The irregularities, moreover, in shape, size and position of ballast particles make it impossible to attain an exact mathematical expression for the transmission of pressures through the ballast. An equation was developed, therefore, that takes into consideration both theory and practice. A theoretical equation based on ideal conditions was first developed and then multiplied by a correction factor based on experimental results. Either one of the following two equations may be used to determine the vertical pressure distribution below and to the right or left of the center line of a tie.

1) Based on the Theory of Elasticity:

From the theory of elasticity, if a portion MN of the boundary of a semi-infinite mass Fig. 3, is loaded with a strip load of an intensity p_a per unit of area, the vertical pressure at point 0 is:

$$p = \frac{C\ p_a}{\pi}\left[\alpha + \sin\alpha \cdot \cos 2\beta\right] \qquad (1 \ \& \ 2)$$

The correction factor, C, is equal to the vertical pressure below the center line of the tie, as obtained by test, divided by the vertical pressure below the center line as determined by the equation. Theory assumes the tie load to be uniformly distributed and the ballast material to be homogeneous and elastic.

2) Based on the geometry of a pressure bulb and the use of the probability curve:

The total pressure per unit length along the longitudinal direction of the tie on plane I-I = total pressure per unit length on plane II-II which passes through the center of the pressure bulb, Figure 4.

$$r^2 = h_o^2 + \left(\frac{b}{2}\right)^2 \qquad (3)$$

where

r = radius of the pressure bulb,

h_o = depth below the tie to the center of the

b = width of the tie.

Total pressure per unit length (along the longitudinal direction of the tie) on plane I-I, which is on top of the ballast = total pressure per unit length on plane II-II, which passes through the center of the pressure bulb. Hence,

$$b\ p_a = 2\ r\ p_o \qquad (4)$$

where

p_a = pressure per unit area on the top of the ballast

$= \dfrac{\text{Tie load}}{b \times \text{length of the tie}}$

p_o = pressure per unit area normal to the surface of a pressure bulb.

From equation 3 we get

$$h_o^{\,2} = r^2 - \left(\frac{b}{2}\right)^2 \qquad (3)'$$

From equation 4 we get

$$r = \frac{b}{2} \cdot \frac{p_a}{p_o} \qquad (4)'$$

Substituting equation 4' into equation 3' we get

$$h_o^2 = \left(\frac{b}{2}\right)^2 \left(\frac{p_a}{p_o}\right)^2 - \left(\frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2 \left[\left(\frac{p_a}{p_o}\right)^2 - 1\right]$$

$$h_o = \frac{b}{2} \sqrt{\left(\frac{p_a}{p_o}\right) - 1} \qquad (5)$$

$$h_1 = r = \frac{b}{2} \cdot \frac{p_a}{p_o}$$

At a depth equals $h_o + h_1$ below the center line of the tie, the vertical pressure in the ballast is p_o.

Hence,

$$h_o + h_1 = h = \frac{b}{2} \sqrt{\left(\frac{p_a}{p_o}\right)^2 - 1} + \frac{b}{2} \cdot \frac{p_a}{p_o}$$

Hence,

$$p_o = \frac{4\, b\, h\, p_a}{4\, h^2 + b^2} \qquad (6)$$

Pressure p_o given by equation 6 is based on assumptions which are not exactly representative to a tie under load, and in order to get a fairly representative pressure, equation 6 is multiplied by a correction factor "K" which is based on test results.

Therefore equation 6 becomes:

$$p_o = \frac{4\ b\ h\ K\ p_a}{4\ h^2 + b^2} \tag{7}$$

where

$$K = \frac{\text{Vertical pressure below the center-line of the tie determined by test}}{\text{Vertical pressure below the center-line of the tie determined by equation 6}}$$

In order to determine the vertical pressure f (x, h) at any point (x, h) in the ballast, where x is the distance to the right or to the left of the center-line of the tie and h is the vertical distance below the bottom of the tie, it is assumed that, for fixed h, f as a function x has the shape of the normal density. That assumption was made by Talbot (12) to develop his equation. That is, we assume

$$f\ (x,\ h) = \frac{N}{\sqrt{2\ \pi} \cdot \sigma}\ e^{-\frac{x^2}{2\ \sigma^2}} \tag{8}$$

where

σ = Standard deviation which is a function of h

and

$N = b\ p_a$

Expressing the dependence of f on x and h, and putting $p = f\ (x,\ h)$, we have then

$$p = \frac{b\ p_a}{\sqrt{2\pi}\cdot\sigma}\, e^{-\frac{x^2}{2\sigma^2}} \tag{9}$$

Substituting $x = 0$ into equation 9 we get

$$p = \frac{b\ p_a}{\sqrt{2\pi}\cdot\sigma} = p_o \tag{9'}$$

From equation 7 and 9' we get

$$\frac{4\ b\ h\ K\ p_a}{4\ h^2 + b^2} = \frac{b\ p_a}{\sqrt{2\pi}\cdot\sigma}$$

and

$$\sigma = \frac{4\ h^2 + b^2}{4\sqrt{2\pi}\cdot h\cdot K}$$

Substituting σ in equation 9 we get

$$p = \frac{4\ b\ h\ K\ p_a}{4\ h^2 + b^2}\, e^{\frac{-x^2\left[4\sqrt{2\pi}\cdot K\ h\right]^2}{2\left[4\ h^2 + b^2\right]^2}}$$

$$p = \frac{4\ b\ h\ K\ p_a}{4\ h^2 + b^2}\, e^{\frac{-50\ K^2\ h^2\ x^2}{\left[4\ h^2 + b^2\right]^2}} \tag{10}$$

or

$$p = \frac{4\ b\ h\ K\ p_a}{4\ h^2 + b^2}\ (10)^{\frac{-21.74\ K^2\ h^2\ x^2}{\left[4\ h^2 + b^2\right]^2}} \qquad (11)$$

For b = 9 inches

$$p = \frac{9\ h\ K\ p_a}{h^2 + 20.25}\ (10)^{\frac{-1.36\ K^2\ h^2\ x^2}{(h^2 + 20.25)^2}} \qquad (12)$$

Conclusions

Under the test conditions of a subgrade compacted to the stated density:

(1) The depth of ballast needed to get a fairly uniform pressure on the subgrade equals the tie spacing minus three inches. The vertical pressure at this depth should be less than the allowable bearing capacity of the subgrade to prevent subgrade deformation.

(2) The magnitude of the vertical pressure below the center-line of a tie is always smaller than that given by Talbot for the same unit pressure applied.

(3) Either one of following two equations may be used to determine the vertical pressure distribution below and to the right and left of the center-line of a tie:

(a) Based on the Theoretical Elasticity:

$$p = \frac{C\ p_a}{\pi}\ [a + \sin \alpha\ .\ \cos 2\ \beta] \qquad (2)$$

(b) Based on the geometry of a pressure bulb

$$p = \frac{9\ h\ K\ p_a}{h^2 + 20.25} \quad (10) \qquad \frac{-1.36\ K^2\ h^2\ X^2}{(h^2 + 20.25)^2} \quad (12)$$

These equations are limited to wood ties of 9 inch widths and to the particular values of the correction factors C and K which were given for depths ranging from 6 to 30 inches.

(4) In order to develop a general equation for p, equation No. 2 was simplified by using an average value of C by relating the correction factor directly to depth by a simple straight line relationship. The following equation is limited to depths ranging from 6 to 30 inches and to wood ties of 9 inch widths.

$$p = \frac{p_a}{\pi}\left(\frac{48 - h}{22}\right)[\alpha + \sin\alpha\ .\ \cos 2\ \beta] \qquad (13)$$

(5) The three types of ballast used in this research behaved in a similar manner as regards the magnitude of vertical pressures.

Table 1. Calculated and Experimental Values of Vertical Pressure At Various Points Below A Single Tie

Tie Load lb.	h Ins.	X Ins.	Calculated Vertical Pressure[1] psi	Calculated Vertical Pressure[2] psi	Calculated Vertical Pressure[3] psi	Experimental Vertical Pressure psi
6,666	6	0	8.65	8.65	12.82	8.65
	12	0	4.50	4.50	5.41	4.50
	12	10.5	1.49	0.86	0.255	1.10
	18	0	2.71	2.71	3.25	2.71
	18	10.5	1.50	1.50	1.08	1.35
	24	0	1.76	1.76	2.26	1.76
	24	10.5	1.22	1.37	1.32	1.17
	30	0	1.02	1.02	1.73	1.02
	30	10.5	0.81	0.92	1.27	0.70
10,000	6	0	14.30	14.30	19.23	14.30
	12	0	7.15	7.15	8.115	7.15
	12	10.5	2.36	1.13	0.385	1.20
	18	0	4.22	4.22	4.875	4.22
	18	10.5	2.33	2.23	1.62	2.00
	24	0	2.68	2.68	3.39	2.68
	24	10.5	1.85	2.05	1.98	1.60
	30	0	1.58	1.58	2.595	1.58
	30	10.5	1.26	1.44	1.91	1.12
20,000	6	0	32.52	32.52	38.46	32.52
	12	0	16.00	16.00	16.23	16.00
	12	10.5	5.30	1.60	0.77	2.63
	18	0	9.40	9.40	9.75	9.40
	18	10.5	5.18	4.20	3.24	4.70
	24	0	5.82	5.82	6.78	5.82
	24	10.5	4.02	4.33	3.96	3.55
	30	0	3.35	3.35	3.19	3.35
	30	10.5	2.66	3.02	3.82	2.36

1. $p = \frac{C\ p_a}{\pi}\ [\alpha + \sin\alpha\ .\ \cos 2\beta]$

2. $p = \frac{9\ h\ K\ p_a}{h^2 + 20.25}\ (10)^{\frac{-1.36\ K^2\ h^2\ X^2}{(h^2 + 20.25)^2}}$

3. $p = \frac{16.8\ p_a}{h^{1.25}}\ (10)^{\frac{-6.05\ X^2}{h^{2.5}}}$

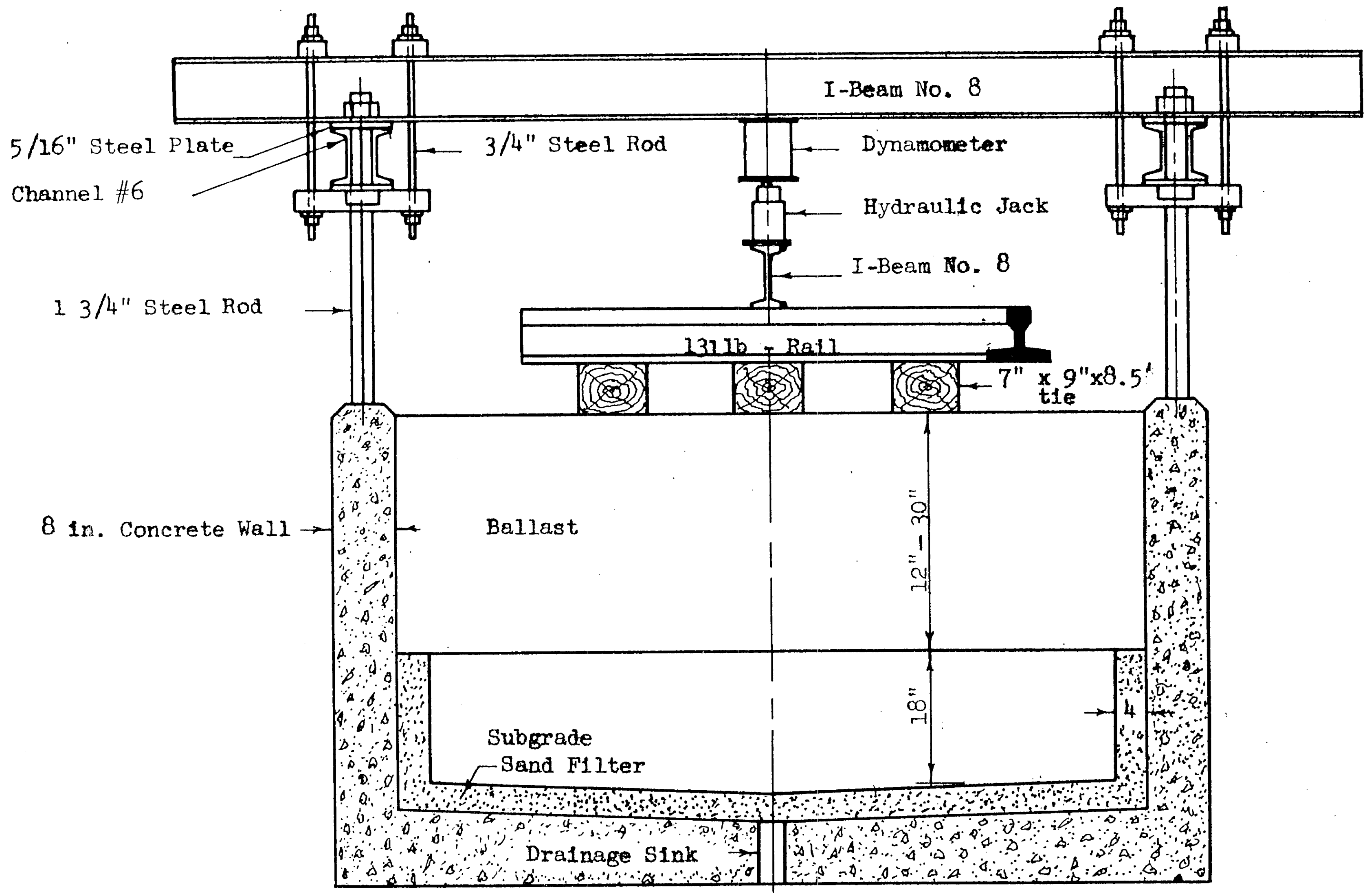
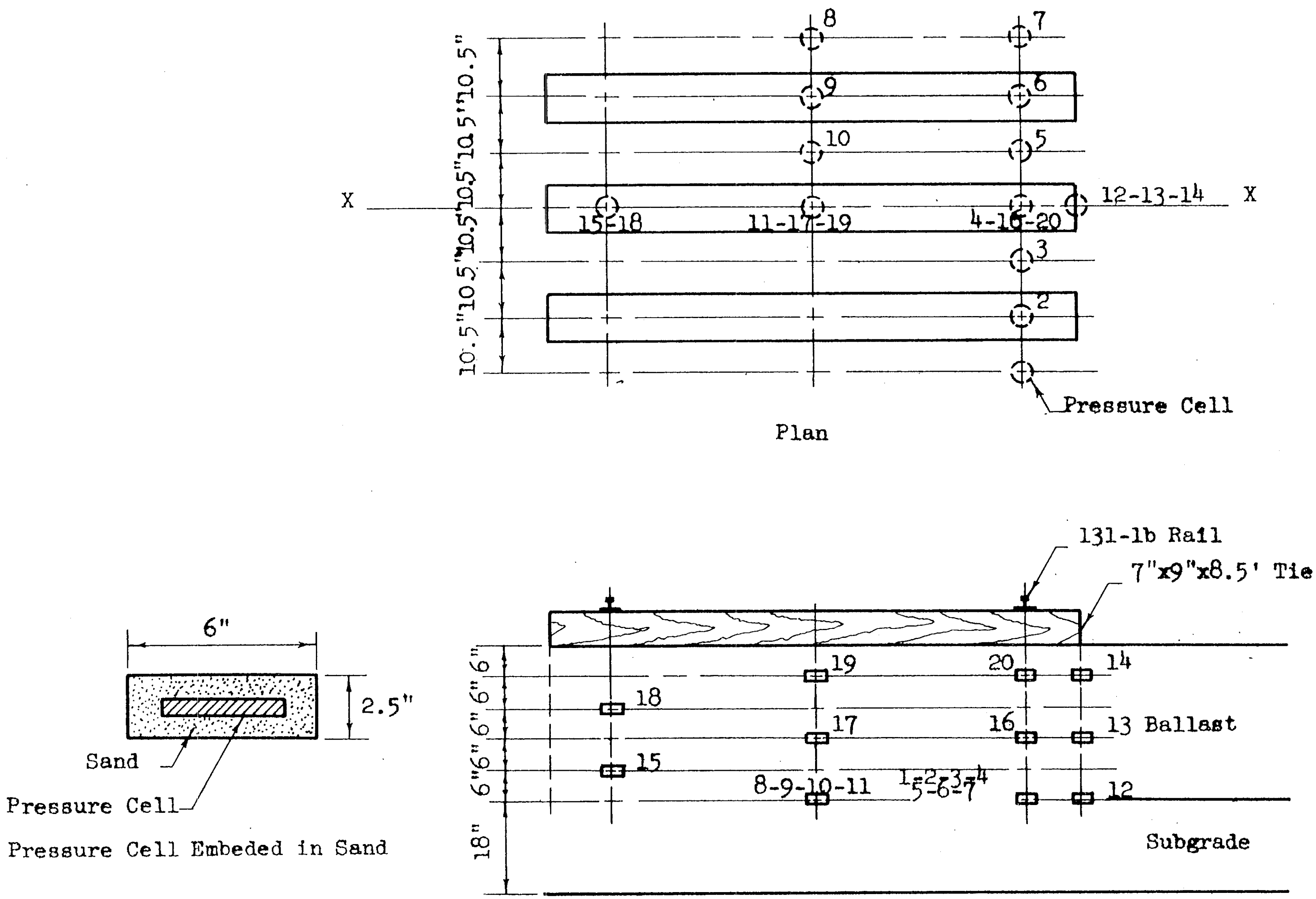


Fig. 1 . Cross Section Of The Test Track



Section Elevation X-X

Fig. 2 . Location Of Pressure Cells Within The Ballast Section

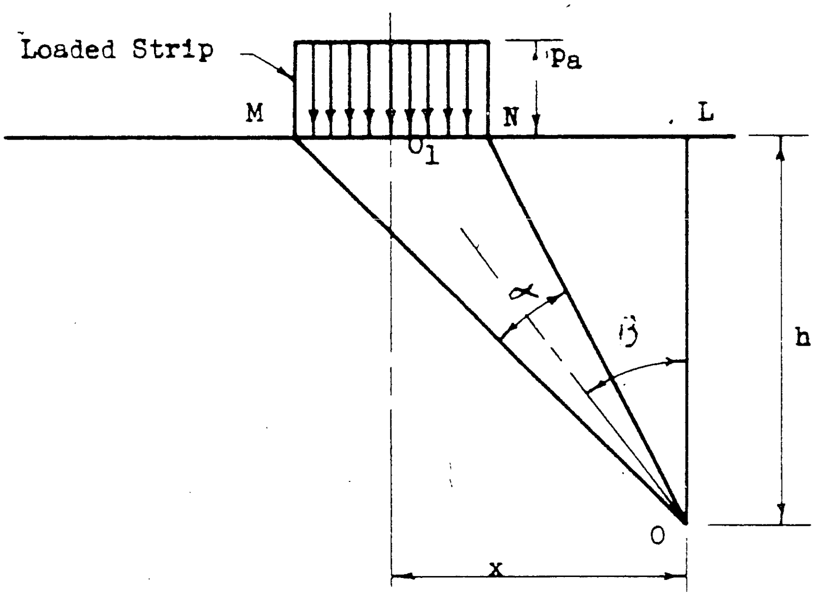


Fig. 3. Coordinate System

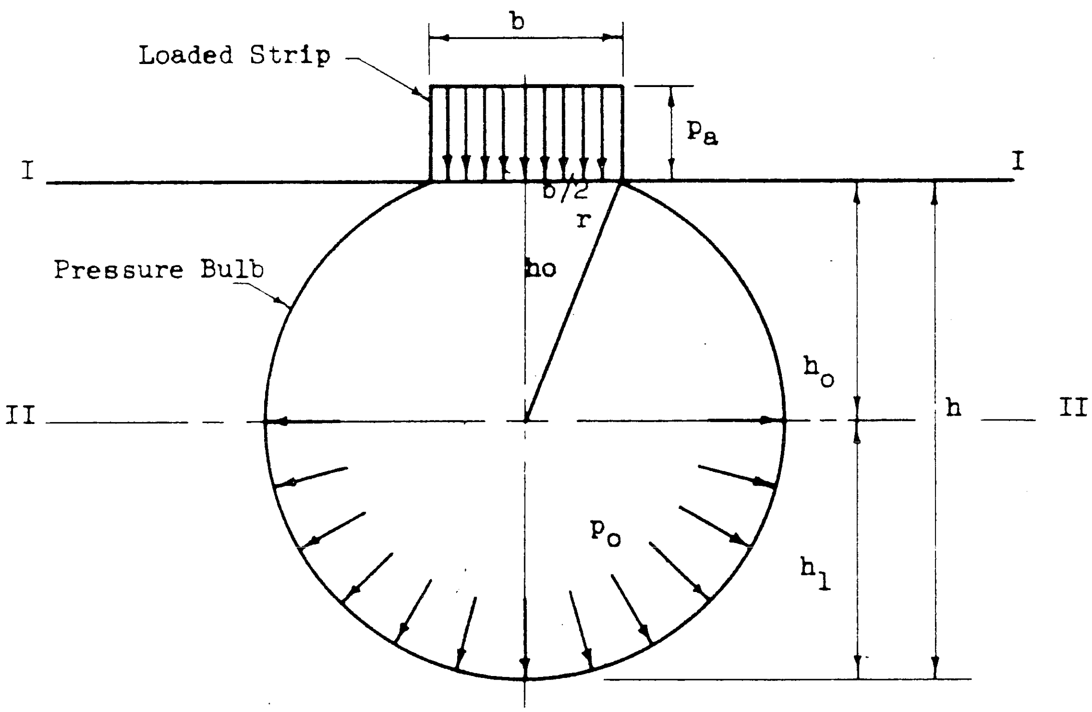


Fig. 4. Geometry of a Pressure Bulb

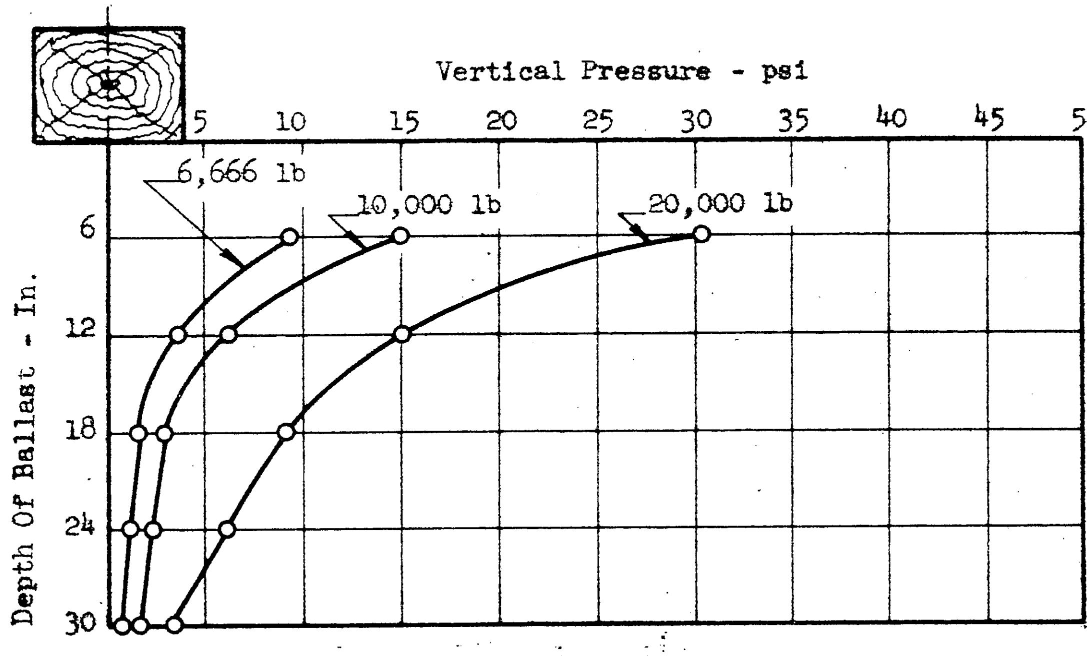


Fig. 5 . Vertical Pressure Distribution At Depths Up To 30 In. Of Crushed Slag Ballast Below The Center-Line Of A Single Tie For Different Tie Loads

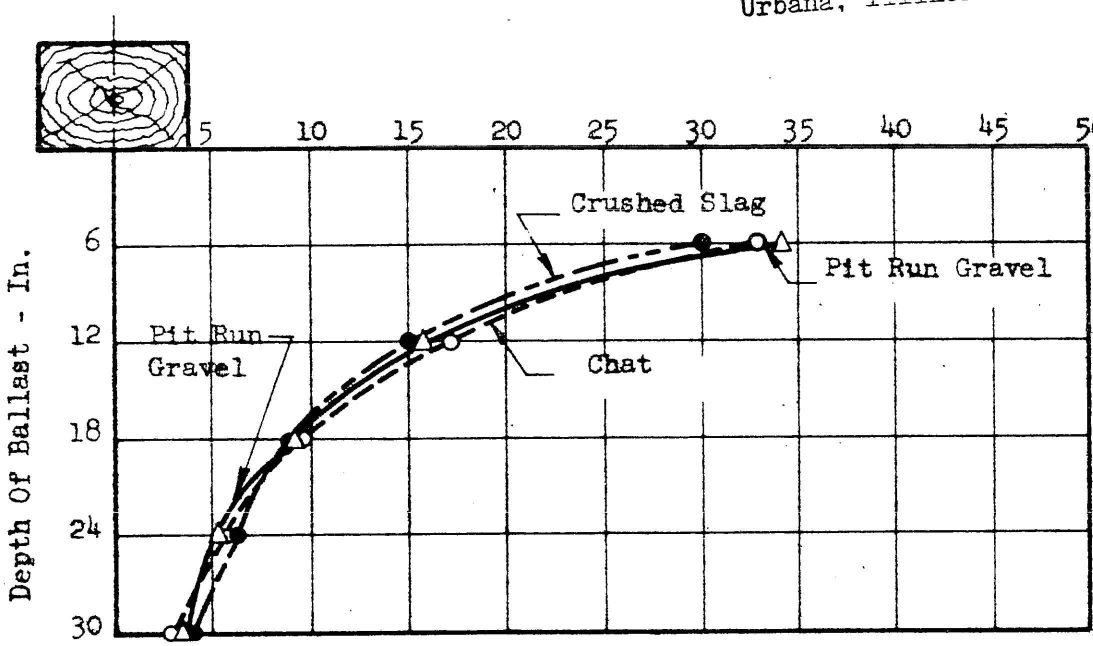


Fig. 6 . Vertical Pressure Distribution At Depths Up To 30 In. Of Ballast Below The Center-Line Of A Single Tie, Tie Load = 20,000 lb

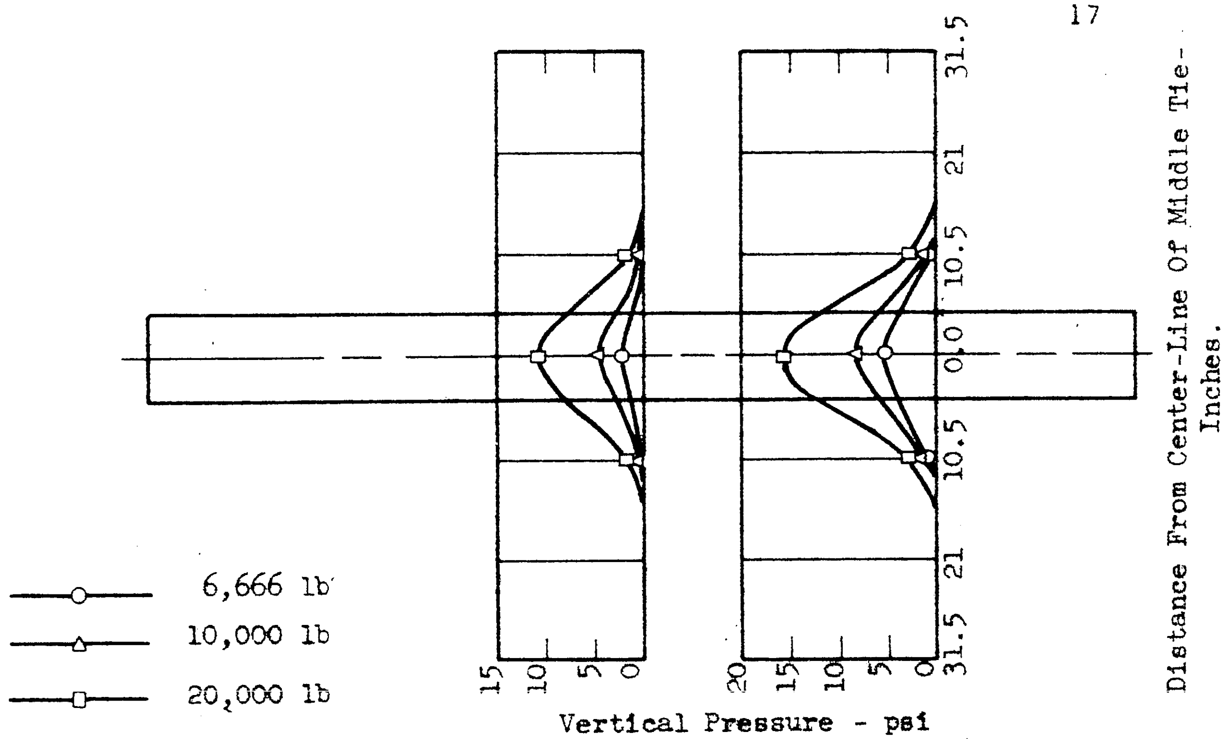


Fig. 7 . Average Vertical Pressure Distribution On The Subgrade At A Depth Of 12 In. Of Ballast Below A Single Tie

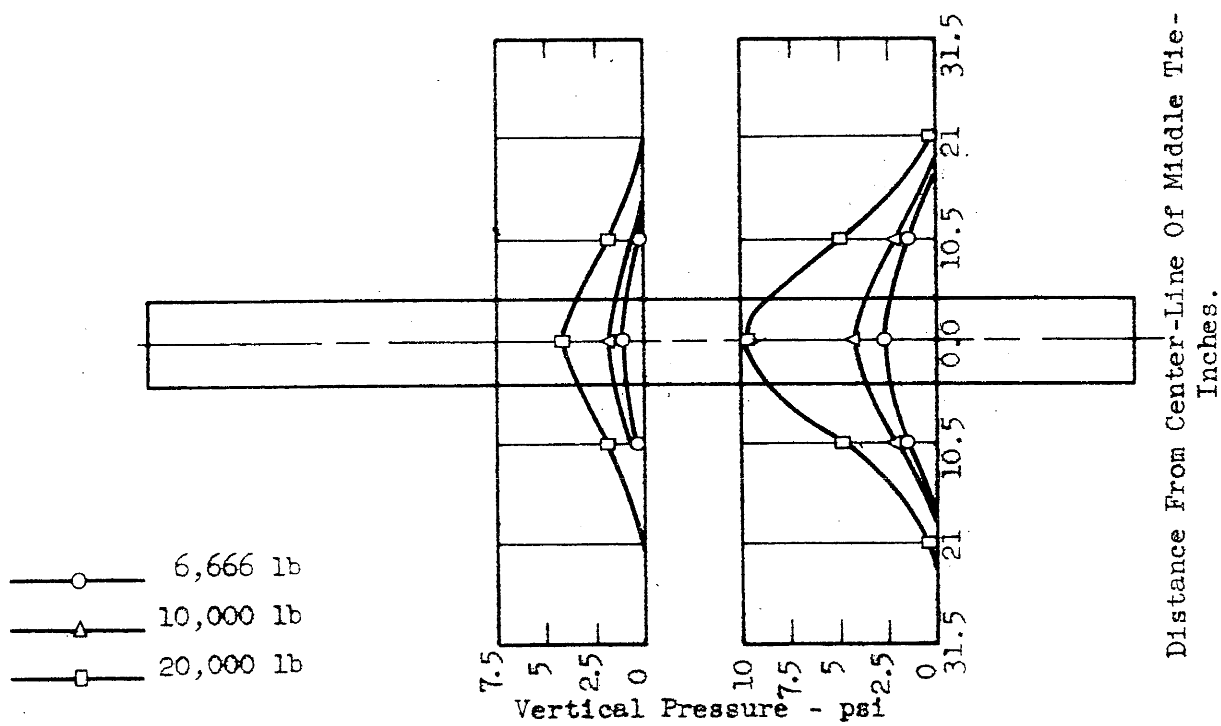


Fig. 8 . Average Vertical Pressure Distribution On The Subgrade At A Depth Of 18 In. Of Ballast Below A Single Tie

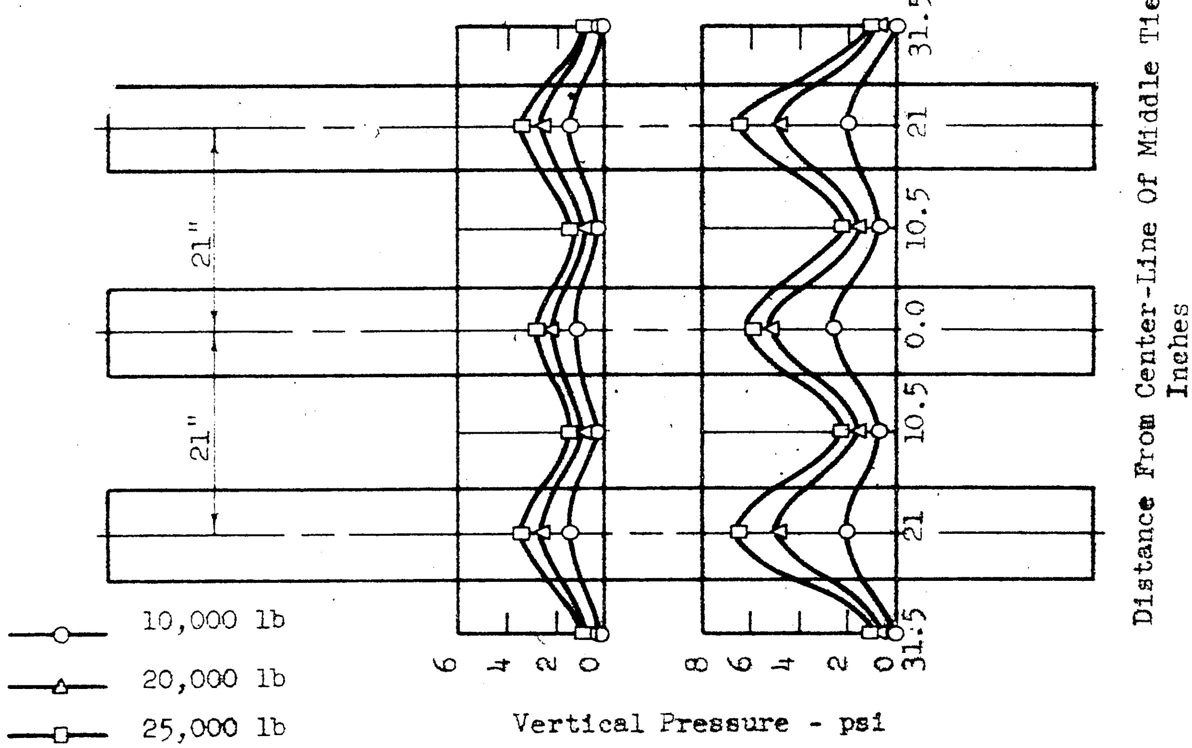


Fig. 9 . Average Vertical Pressure Distribution On The Subgrade At A Depth Of 12 In. Of Ballast Below Three Ties

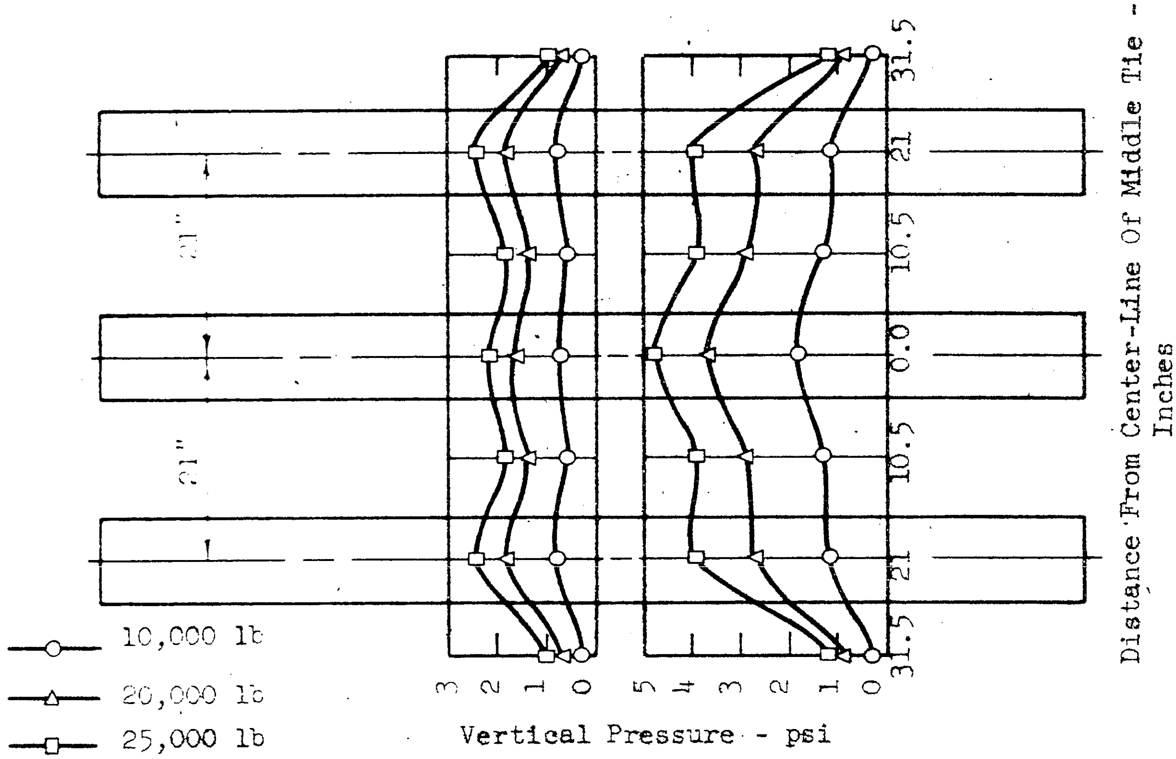


Fig. 10 . Average Vertical Pressure Distribution On The Subgrade At A Depth Of 18 In. Of Ballast Below Three Ties

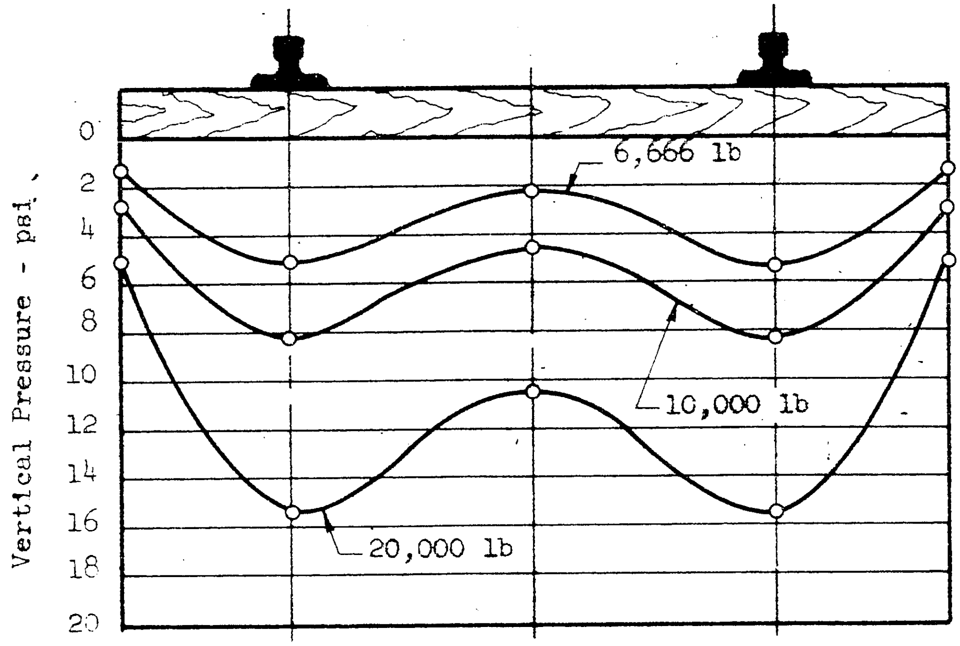


Fig. 11. Average Vertical Pressure Distribution On The Subgrade At A Depth Of 12 In. Of Ballast Below A Single Tie

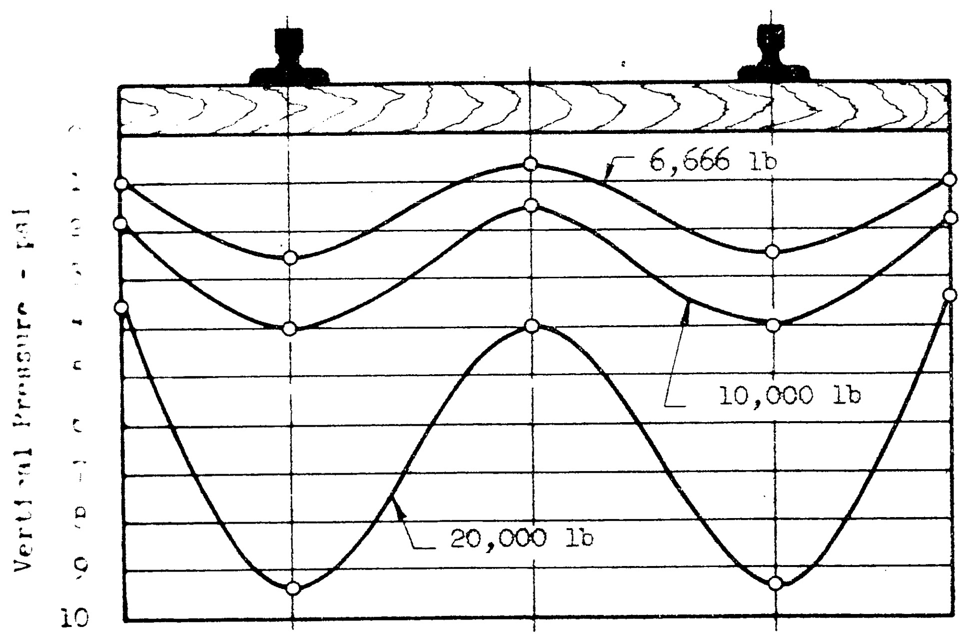


Fig. 12 . Average Vertical Pressure Distribution On The Subgrade At A Depth Of 18 In. Of Ballast Below A Single Tie

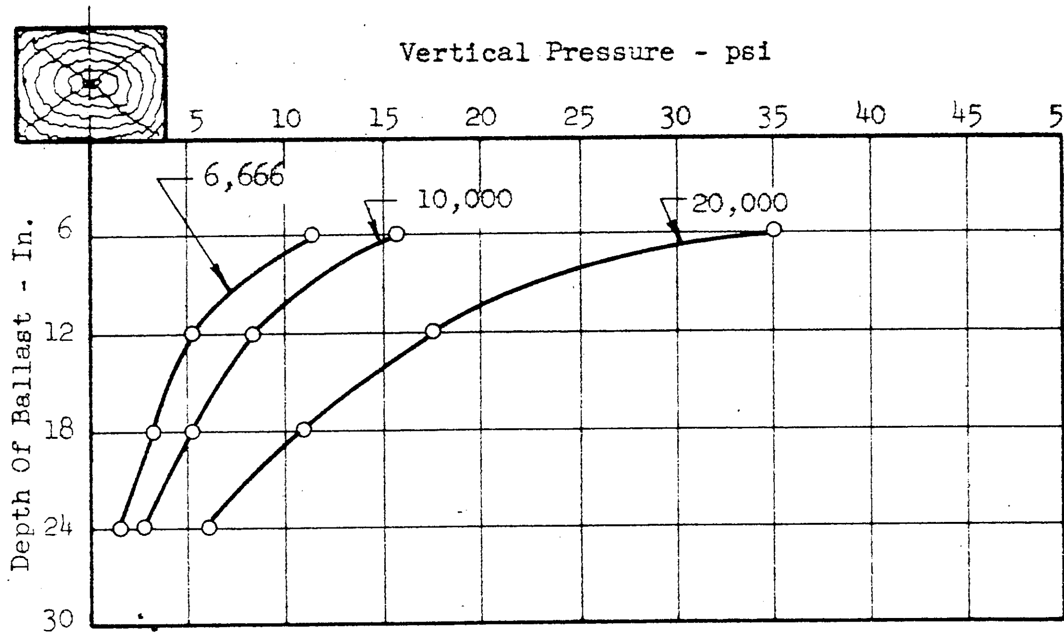


Fig. 13. Average Vertical Pressure Distribution At Depths Up To 24 In. Of Ballast Below The Center-Line Of A Single Tie

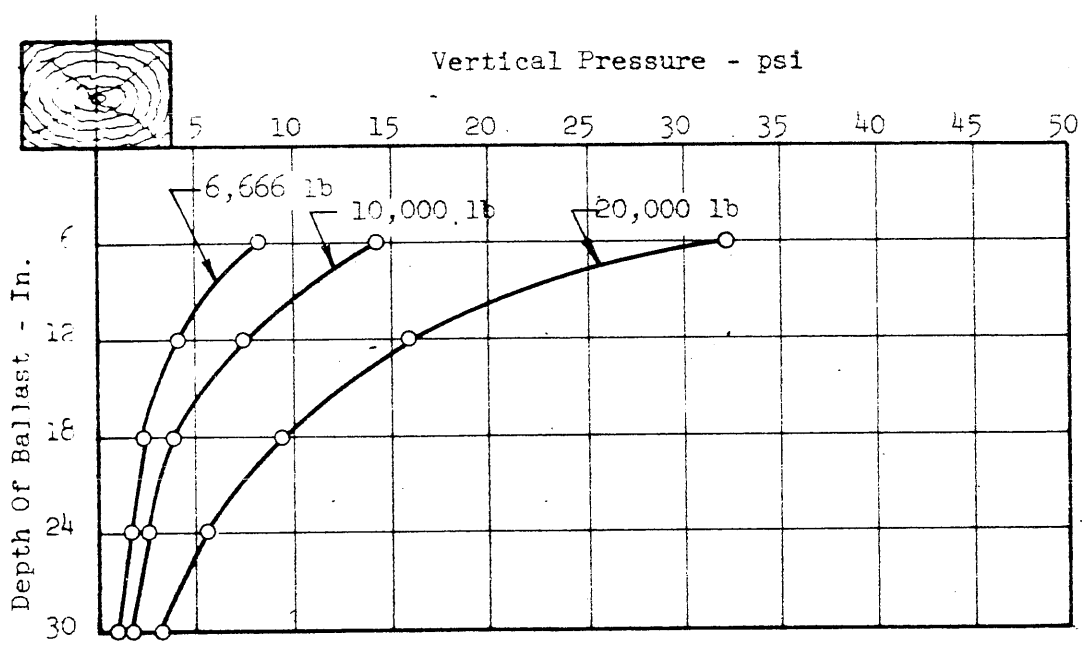


Fig. 14. Average Vertical Pressure Distribution At Depths Up To 30 In. Of Ballast Below The Center-Line Of A Single Tie

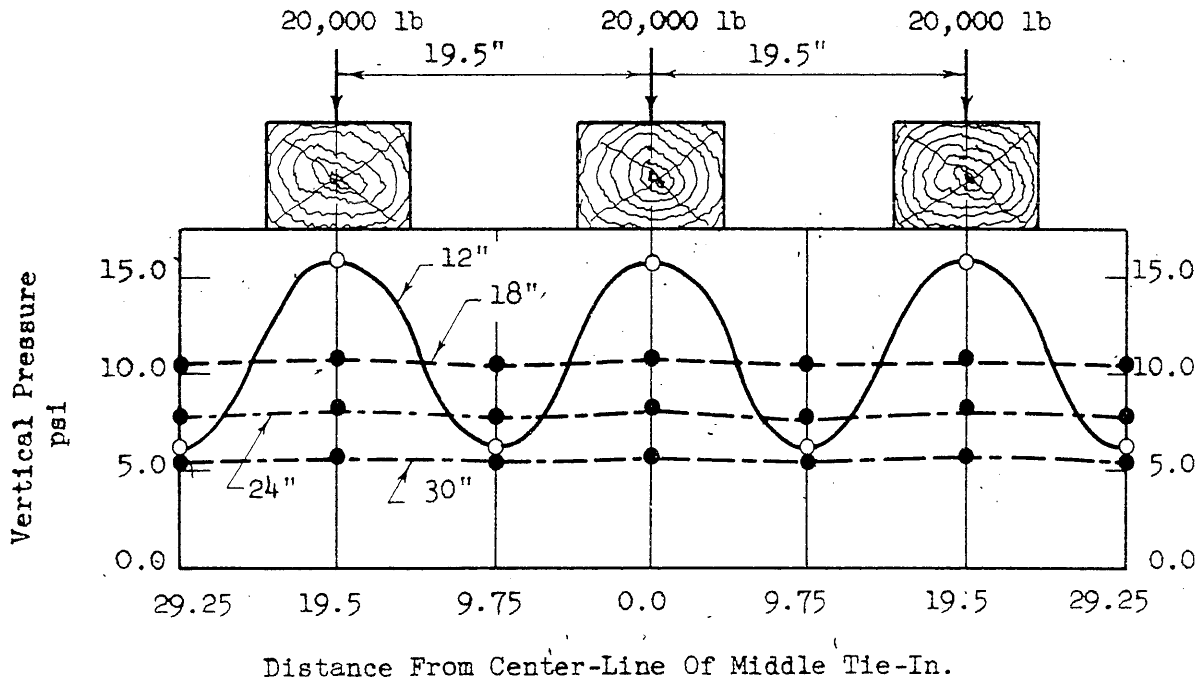


Fig. 17 . Average Vertical Pressure Distribution On The Subgrade Using The Principle Of Superposition For Different Depths Of Ballast At A Tie Spacing Of 19.5-In.

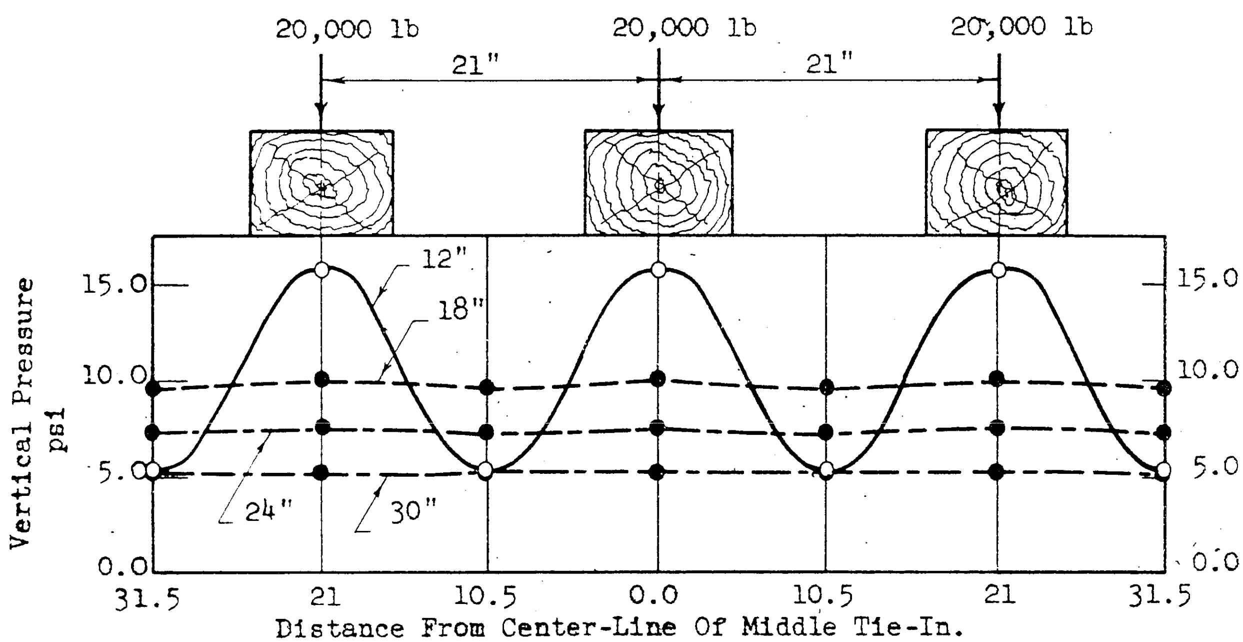


Fig. 18 . Average Vertical Pressure Distribution On The Subgrade Using The Principle Of Superposition For Different Depths Of Ballast At A Tie Spacing Of 21-In.

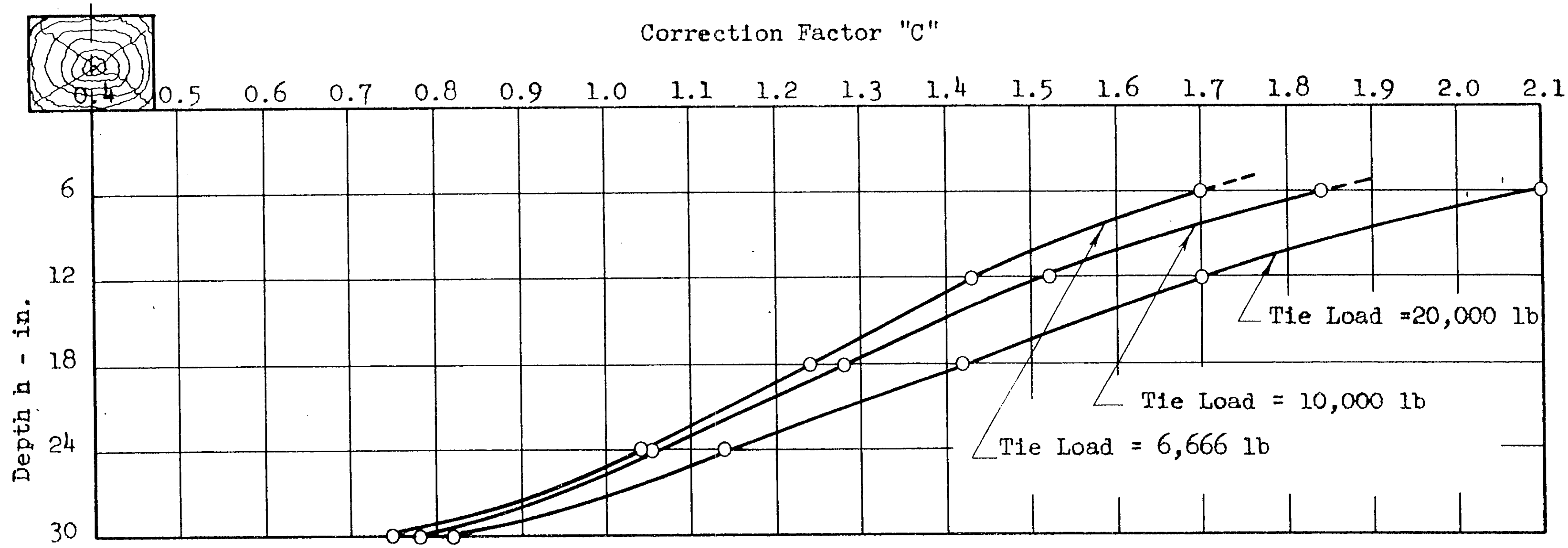


Fig. 19 . Relation Between Correction Factor "C" And Depth "h" For Different Tie Loads