INTERIM REPORT
MOISTURE MOVEMENT AND MOISTURE EQUILIBRIA IN PAVEMENT SYSTEMS

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by
BARRY J. DEMPSEY
and
ATEF ELZEFTAWY

A Report of the Investigation of
Moisture Movement and Moisture Equilibria in
Pavement Systems
Project IHR-604
Illinois Cooperative Highway and Transportation Research Program

conducted by the
TRANSPORTATION RESEARCH LABORATORY
DEPARTMENT OF CIVIL ENGINEERING
ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

in cooperation with the
STATE OF ILLINOIS
DEPARTMENT OF TRANSPORTATION
and the
U.S. DEPARTMENT OF TRANSPORTATION
FEDERAL HIGHWAY ADMINISTRATION

UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS
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Abstract:
The primary purpose of IHR-604 is to develop a satisfactory and realistic procedure for determining moisture movement and moisture equilibria in pavement systems and to develop procedures for utilizing this information in pavement design. The specific objectives of this interim report were to fulfill the requirements of Phase I (Development of Working Moisture Model), as follows:

1. Part 1.1 - Develop, based on environmental factors, a theoretical model for predicting moisture movement and moisture equilibria in pavement systems.
2. Part 1.2 - Validate the moisture model and if necessary refine the moisture model by means of controlled laboratory studies.
3. Part 1.3 - Prepare the interim report concerning the development of the moisture model and its validation by using laboratory data and available field data.

It is expected that the findings from this research project will be helpful to those who must make decisions relating to moisture problems in pavement systems. The moisture model can be used to determine how various design modifications influence the moisture regime in a pavement system. It is anticipated that the model and related field and laboratory studies will lead to a less empirical approach for incorporating moisture effects into pavement design, construction, behavior, and performance.

Keywords: Moisture Movement, Subsurface Drainage, Moisture Model, Environmental Effects, Drainage

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INTRODUCTION

1.1 General

Project IHR-604, "Moisture Movement and Moisture Equilibria in Pavement Systems," was initiated on November 1, 1971 to study moisture conditions in pavement systems. The engineering problems associated with the behavior of soils and pavement materials responsive to moisture changes indicated that further study was required with reference to both the methods of predicting moisture changes and the engineering consequences of such moisture changes.

The migration of moisture in the subgrade beneath a pavement surface can cause considerable volume change in the soil and possibly in the pavement system as a whole. Volume change in airport pavements caused by frost action is a primary cause of pavement damage in cold climates. Moisture movement through the soil to a freezing zone in response to a temperature gradient often causes ice lenses to form which cannot be accommodated by the pore spaces in the soil. The end results are heave during freezing and loss of strength and stability during thawing or a durability type failure in the pavement system. Shrinkage and swell problems in many fine-grained subgrade soils are also attributed to moisture changes.

Moisture along with temperature has a major influence on the performance of pavement surfaces. Moisture induced surface deterioration and failure in portland cement concrete pavements and asphalt concrete pavements are major causes of pavement closures and maintenance. Surface failures in many pavement systems also occur because of loss of subgrade strength caused by moisture changes or frost action.

1.2 Project Objectives

The purpose of Project IHR-604 is to develop a satisfactory and realistic procedure for determining moisture movement and moisture equilibria in pavement
systems and to develop procedures for utilizing this information in pavement design.

The specific objective to be achieved in the project are as follows:

A. Phase 1: Development of Working Moisture Model

1. Part 1.1 - Develop, based on environmental factors, a theoretical model for predicting moisture movement and moisture equilibria in pavement systems.

2. Part 1.2 - Validate the moisture model and if necessary refine the moisture model by means of controlled laboratory studies.

3. Part 1.3 - Prepare an interim report concerning the development of the moisture model and its validation by using laboratory data and available field data.

B. Phase 2: Moisture Movement and Moisture Equilibria Studies

1. Phase 2.1 - Conduct field studies of the moisture regime in selected pavement systems and obtain data for further validation and, if necessary, refinements of the moisture model.

2. Phase 2.2 - Utilize the moisture model to determine how parameters such as water table depth, external drainage conditions, material properties, and climate influence the moisture regime in various pavement systems.

3. Part 2.3 - Analyze data from the theoretical studies, laboratory studies, and field investigations.

C. Phase 3: Summarize the study findings and make recommendations to the sponsoring agencies regarding the practical application of the findings and utilization of the moisture model in pavement design.
This interim report summarizes the Phase I research to develop a working moisture model.

1.3 Significance of Work

Research at the University of Illinois has indicated that moisture is an important factor affecting the durability properties and resilient properties of highway soils and materials, and the performance and fundamental behavior of pavement systems. Water directly governs the mechanical properties of most pavement materials and soils, therefore, any variation in water content will alter the properties of most pavement materials and soils. From materials research it has become apparent that methods for predicting moisture movement and moisture equilibria in pavement systems are needed in order to fully describe material and pavement behavior. The increased use of subsurface drains also supports the need for a better understanding of moisture movement and moisture equilibria in pavement systems. It is evident that sound decisions concerning the location of subsurface drains require a thorough knowledge of time-dependent moisture regime of the pavement system.

The importance of research to study moisture conditions in pavement systems is reflected in many ways. During 1973 the Federal Highway Administration conducted five workshops on "Water in Pavements." The purpose of these workshops was to gain a better understanding of the effects of moisture on pavement systems. In an international report published in 1973 entitled "Water in Roads," a research group chaired by the Organization for Economic Co-operation and Development (OECD) discussed the importance of predicting and controlling moisture in pavement systems. The OECD (1) recommended that the procedures shown in Figures 1.1, 1.2, and 1.3 be used to develop research for predicting and controlling moisture in pavement systems. The research objectives and procedures of IHR-604 are in agreement with those recommended by OECD (1).
It is expected that the findings from this research project will be helpful to those who must make decisions relating to moisture problems in pavement systems. The moisture model can be used to determine how various design modifications influence the moisture regime in a pavement system. It is anticipated that the model and related field and laboratory studies will lead to a less empirical approach for incorporation moisture effects into pavement design, construction, behavior, and performance.
2.1 General

The prediction of the moisture conditions in a pavement system for a given time, climate, and topographical location is a complicated matter. The multitude of the variables involved tend to limit the conclusiveness of both the field measurements and the rational or theoretical type of approach. Although there are deficiencies in the present knowledge of moisture movement and moisture equilibria in pavement systems, substantial understanding of moisture conditions in soils has been gained from research in hydrology, agriculture, and soil science. It is evident that moisture research contributions made by other fields will aid in the study of moisture in pavement systems.

Before undertaking the task of predicting moisture conditions in pavement systems, it would be wise to heed the words of Winterkorn (2) who wrote the following:

"Water and soil are not only the most important materials in this world, but also the most complex ones, each in its own category, water as a liquid and soil as a multi-phase dispersed system. As a matter of fact, this very complexity of water and soil is the reason for this importance in engineering as well as biology. If water and soil are combined to moist soil systems, then the complexity of each is multiplied by that of the other. At this point, the coward gives up, the conscientious scientist and engineer goes on. He observes then describes his observations, later, he classifies and systematizes facts found by himself and others, mechanisms can be recognized and, if they are simple enough, be described by means of mathematical formulae. The total complexity, however, remains and any theory or hypothesis that is sufficiently simple to permit mathematical formulation is either so general as to be obvious, or is applicable only to a limited area of the total field."

The engineer should not hope for an exact solution to the moisture movement problem, but a series of approximations that will yield acceptable values
as output. However, notice that the civil engineer, unlike the soil physicist, should not consider the prediction of moisture movement an end in itself. It should be considered a tool to be used for optimization of the pavement system, and for the economical and ecological benefits that would be derived from that process.

This phase of study will be concerned with the following subject areas:

1. Moisture conditions in pavement systems.
2. Equilibrium in soil-water systems.
3. Flow in soil-water systems.
4. Methods of predicting moisture conditions in pavement systems.

2.2 Moisture Conditions in Pavement Systems

Low and Lovell (3) have indicated that moisture in pavement systems can come from several sources, Figure 2.1. They have generalized the concept of the source of pavement moisture as follows:

1. Moisture may permeate the sides, particularly where coarse-grained layers are present or where surface drainage facilities within the vicinity are inadequate.
2. The water table may rise (this can be expected in the winter and spring seasons).
3. Surface water may enter joints and cracks in the pavement, penetrate at the edges of the surfacing, or percolate through the surfacing and shoulders.
4. Water may move laterally.
5. Water may move vertically in capillaries or interconnected water films.
6. Moisture may move in vapor form, depending upon adequate
temperature gradients and air void space.

It can generally be concluded that one or more of the above mechanisms
will influence moisture changes in pavement systems.

Most studies of moisture conditions in pavement systems indicate that
moisture content varies with seasons, climatic conditions, geographic loca-
tion, and type of pavement system. Based on a study of thirteen United States
airfields located in non-frost action areas, Redus (4) indicated that mois-
ture conditions may go up or down following construction and appear to stabi-
alyze after about 2 years, subsequently displaying small fluctuations (increases
and decreases). Low and Lovell (3) concluded from their study of current
literature that the moisture contents show continuous, if small, variation
with the seasons. In a long term study of moisture conditions beneath rigid
pavements in Missouri, Guinnee and Thomas (5) found that moisture variations
in the top levels of the subbase and subgrade were greater than those noted
at deeper levels. Chu and Humphries (6) found that moisture contents in pave-
ment systems in South Carolina varied with season, soil type, and location in
the pavement system, Figure 2.2. They also found that the subgrade moisture
content was influenced by the depth of the ground water table, Figure 2.3.

Atchison and Richards (7), based on results from their study of moisture
conditions in pavement subgrades throughout Australia, stated that moisture
stability beneath the greater part of the paved area was similar for every
test site even though the climatic conditions at the sites were widely dif-
f erent. A field study conducted by the Corps of Engineers (8) indicated that
the amount of precipitation may have considerable influence on the moisture
conditions in pavement subgrade and base course materials, Figure 2.4.

Marks and Haliburton (9), based on a short term study of typical Oklahoma
highways, concluded that most moisture variations occur beneath highway pavements
on an annual cycle with maximum moisture contents occurring during the winter months. Hicks (10) found that for 20 stations in North Carolina the moisture contents for the bases and subgrades were highest during late winter or early spring.

Frost action effects accentuate moisture content increases in the pavement system. Road Research Laboratory studies indicate a significant increase (1 to 3 percent) in granular base moisture content due to frost action (11). Stevens, Maner, and Shelburne (12) observed that spring pavement break up on Virginia could be related to the amount of fall precipitation and the length of the freezing period. They found that large amounts of precipitation in October, November, and December tended to saturate the subgrade and base beneath the highways studied. Straub, Dudden, and Moorhead (13) found significant moisture increases beneath snow covered shoulders during periods of thawing as a result of gravitational flow of snowbank melt water.

Not all authors agree on the extent to which precipitation influences the variation in subgrade moisture content. Kubler (14), after analyzing much data on subgrade moisture content and precipitation for West Germany, could not establish a relationship between precipitation and the change in subgrade moisture content. He found that precipitation before freezing, during intervening thaw periods, and during the final spring thaw had no influence on the degree of frost action.

Marks and Haliburton (9) indicated that cyclic moisture content variations were considerably affected by precipitation at sites where the pavements were poor (greater degrees of cracking and perviousness in the pavement surface). Moisture content changes could not be correlated with precipitation for pavement sections with high ratings (little cracking, good surface
condition). The moisture content variations for the higher rated pavements were primarily attributed to temperature effects.

The results of an analysis of the influence of precipitation on soil moisture contents by Moulton and Dubbe (15) showed that the amounts of precipitation occurring at various periods prior to moisture content sampling were not statistically significant in explaining the observed variations of moisture content in either the base and subbase materials or in the subgrade soils. They felt that the moisture content in granular base and subbase materials would be more dependent upon the drainage characteristics of the materials and the site than upon precipitation.

Observations by Turner and Jumikis (16) showed that moisture content beneath a pavement was affected by the ground water table. They found that precipitation could modify the position of the water table and subgrade moisture content and that the degree of change was sensitive to the type of precipitation. They found that more water from melting snow precipitation percolated into the ground than if the precipitation was in the form of rain.

Several investigators (4, 5, 17) have concluded that moisture contents at the pavements edges are generally higher than those at the interior locations. Guinnee and Thomas (5) point out that water enters the pavement more easily and in greater volumes at the pavement edges. Benkelman (18) in an analysis of WASHO Road Test deflection data correlated inner and outer wheel path deflections with degree of subgrade saturation, as shown in Figure 2.5. According to Benkelman (18), adverse moisture conditions existed at the pavement edges. Atchison and Richards (7) also noted the greater moisture content fluctuation at the pavement edges for Australian pavements.

Moisture content, dry density, CBR, and plate bearing data were obtained for the flexible pavements in Loop 1 (no traffic) and Loops 3, 4, 5, and 6 at
the AASHO Road Test. The data presented in Figures 2.6, 2.7, and 2.8 indicate moisture content increases and CBR decreases in the base and subbase layers (19). In general the moisture content of the subgrade, Figure 2.9, was quite stable but some decrease in CBR was noted in the spring. It was noted that the increase in CBR strength, spring to summer, of the embankment soil could not be satisfactorily explained by differences in moisture content or density. However recent resilient modulus studies by Robnett and Thompson (20) indicate that 1 or 2 percent moisture content change can have considerable influence on the strength of the AASHO Road Test subgrade soil. Figure 2.10 shows the effect of compaction moisture content variation on the resilient modulus of the AASHO Road Test A-6 subgrade soil and several other soils.

Kersten (21), in his study of airfield subgrade moisture, indicated that there was a slight increase in moisture content with an increase in depth, the average difference between the subgrade surface and the 30 in. depth being only 1.0 to 1.5 percent. Data from Marks and Haliburton (9), Figure 2.11, suggest a rather erratic moisture content-depth relationship. Depending on the nature of the soil profile beneath the pavement, it is possible to have wide variations in field water content because of the differences in equilibrium moisture-tension relationships for different soils.

Kersten (17) has summarized extensive field moisture data for the upper 6 in. of the subgrade near the interior of flexible pavement systems as follows:

1. The degree of saturation existing in the subgrades of numerous projects in six states averaged 73 percent, the general range being from 60 to 81 percent. Fifteen percent of the tests showed a saturation value of 90 percent or higher.

2. The subgrade soils of projects on which a high average percent of saturation occurred were in most instances either clay or silty clay.

3. Saturation percentages vary with soil texture, in general, they are high for the clays and become progressively less for the clay loams, the loams, and the sandy loams.
4. Soils of groups A-6 and A-7 (Public Roads Administration classification) attain higher average percentages of saturation than those of groups A-1, A-2, and A-4.

5. Subgrade moistures expressed as percentages of the plastic limit for a large variety of soils in six stages averaged 77 percent, averages for individual states varied from 64 to 82 percent. Approximately 17 percent of the determinations disclosed moisture contents in excess of the plastic limit.

6. The fine textured soils, such as clays, exhibited a marked tendency to attain moisture contents in excess of their plastic limit. Sandy loams rarely had moisture contents as great as their plastic limit. Loessial silty soils tend to attain moistures close to their plastic limit.

7. The optimum moisture contents of the soils were exceeded by the field moistures in about a third of the determinations reported. Clay soils exceeded the content most commonly but soils of all textures, including the sandy loams, have moistures greater than this value in a substantial proportion of tests.

8. Only slight changes in moisture content for periods of from 1 to 5 years were indicated in tests on several projects. Soils on most of these projects, at the time of the initial tests, already had moisture contents approaching the plastic limits.

9. Clay soils with high percentages of saturation were encountered in areas with annual precipitations as low as 14 in. Most tests in such regions, however, give relatively low saturation values.

Kersten (21) analyzed moisture conditions in both rigid and flexible pavements on airfields in eight Corps of Engineer Districts in the United States. These airfields were located in the Southeast, Midwest, and Southwest. Kersten (21) made the following observations:

1. In humid regions sands and loamy sands had low average percentages of saturation with less than 5 percent as high as 90 percent saturated. As little as 3 percent and as high as 60 percent of the sandy loams in the various districts were 90 percent or more saturated; such soils are apparently quite sensitive to controlling influences and their conditions may reflect weather conditions at the time of construction. The saturation values of the heavier textured soils ran progressively higher with from 30 or more than 50 percent of the individual test results exceeding 90 percent saturation.

2. A similar trend occurred for the comparisons to the plastic limits, the loamy sands never exceeding it, the sandy loams being somewhat variable with an average of 10 percent in excess and the heavier soils showing larger excessive values up to about two-thirds of the clays.
3. The moisture content of about 25 percent of the sands, 70 percent of the sandy loams and 90 to 100 percent of the heavier soils exceeded the optimum moisture content. In arid or semi-arid regions saturation values were not particularly high for any of the textural classes of soil; less than 5 percent of values were in excess of the plastic limit occurred for about 5 percent of the sandy loams and 10 percent of the heavier soils. Six percent of the sands and loamy sands, 40 percent of the sand loams, and close to 60 percent of the heavier soils had moisture which exceeded the optimum moisture content.

4. The airport data of the arid regions were supplemented by tests made in highways adjacent to airfields with pavements of an average age of 12 yr. These tests showed the sandy loams to be extremely dry, only 2 percent being wetter than their plastic limit, the heavier soils exceeded this constant in only a limited number of tests.

5. The moisture conditions for distinctive soil areas, such as the fine coastal sands, the Black Belt clays of Alabama, and the loessial silty soils were generally quite uniform and well defined.

6. Comparisons of the moisture contents in similar soils beneath rigid and flexible pavements on the same airfield showed, in most instances, higher values for the concrete; on the average the percentages of saturation differed by about 10 percent for the two types of pavement.

7. Studies of moisture variations with depth in the subgrade for arid regions were not conclusive but indicated only slight average variations in the upper 3 ft., in general less than 2 percent of moisture, expressed as a percent of the dry weight of soil. In some instances the soil was wetter immediately below the base than at 2 or 3 ft., in other cases the reverse was true.

Based on an extensive study of moisture conditions under flexible airfield pavements, Redus (4) made the following observations.

1. In-place moisture contents varied directly, although erratically, with the percentage passing the No. 200 sieve.

2. In-place moisture contents of base courses and subgrades were always below the liquid limit, and those for base courses were always below the plastic limit. Some subgrade moisture contents exceeded the plastic limit.

3. Moisture contents tended to vary directly with the water-plasticity ratio, which is defined as (moisture content - plastic limit) / (liquid limit - plastic limit) for both base courses and subgrades.
4. The degree of saturation did not correlate with annual rainfall, but did appear to be related to the plasticity of the material. High degrees of saturation (85 percent or more) were found in materials of higher plasticity, but moderate or low degrees were found generally in slightly plastic or nonplastic materials.

The extreme variability that is found in moisture data indicates that effort to correlate in-situ moisture content with various soil parameters provides only a rough estimate of moisture conditions in pavement systems. It is evident that reliable approximations of field moisture conditions must consider the important moisture movement mechanisms shown in Figure 2.1 as well as factors such as climate, geographic location, pavement materials, and pavement geometry. It is also evident that accurate characterization of materials used in the design and construction of airport pavement systems requires proper evaluation of the moisture regime.

The Organization for Economic Cooperation and Development (OECD) (1) has indicated that methods for predicting water content and its fluctuations in pavement systems and methods of controlling moisture content in pavement systems must be investigated simultaneously. The OECD (1) further indicated that the development of methodologies for controlling moisture contents will rely on the ability to accurately predict the moisture regime in pavement systems.

2.3 Pavement Distresses Related to Moisture

The pavement distresses related to moisture are quite numerous. In a report prepared for the Department of the Army Construction Engineering Research Laboratory, Barenberg, Bartholomew, and Herrin (22) identified many of the common types of pavement distresses. Some of the distresses related to moisture and temperature either singularly or in combination are as follows:

1. Flexible pavement systems
   a. Potholes
b. Loss of cover aggregates
c. Raveling
d. Weathering
e. Contraction cracking
f. Alligator cracking
g. Reflective cracking
h. Shrinkage cracking
i. Shoving
j. Frost heave

2. Rigid pavement systems
   a. Faulting
   b. Joint failure
c. Pumping
d. Corner cracking
e. Diagonal cracking
f. Longitudinal cracking
g. Blow-up or buckling
h. Curling
i. D-cracking
j. Surface spalling
k. Steel corrosion

Cedergren (23) has observed similar types of distresses in taxiways and runways of numerous airfields in the United States. Johnson, Berg, Carey, and Kaplan (24) also observed many of the distress manifestations in flexible and rigid pavement systems.

Thompson (25) has cited several examples of how temperature and moisture influence the behavior and performance of pavement systems.
Extensive studies of flexible pavement sections by the Canadian Good Roads Association (26) have shown that the average maximum spring rebound deflection value is equal to 1.63 times the average fall rebound value. In some cases, the spring values were 5.2 times larger than the fall values. Figure 2.12 shows the deflection history of a pavement in Minnesota (27). The pavement section was 3 in. of asphaltic concrete surface, 3 in. of crushed rock base, and 9 in. of sand-gravel subbase. From Figure 2.12 it is quite apparent that temperature and moisture have considerable influence on pavement deflection. The approximate Benkelman Bean rebound deflection for an 18 kip axle load can be calculated by multiplying the Dynaflect values in Figure 2.12 by 20.

Extensive plate loading field test studies for several states in freezing zones have shown that spring bearing values are substantially less than fall values (28, 29, 30, 31). Although the data displayed substantial variability, for many of the flexible pavement sections the spring values were as low as approximately 40 percent of the fall values. The load carrying capacity (30 in. plate, 0.1 in. deformation) of concrete runways studies by Linnel and Haley (32) were reduced, during the spring frost melting period, to approximately 70 percent of the normal load carrying capacity.

Ring (33) has indicated that spring subgrade bearing strengths in the northern states range from 30 percent to 100 percent of the fall values, Figure 2.13.

Benkelman (34) reported that a great deal more structural deterioration took place during the spring months than during the summer months in the AASHO Road Test flexible pavements. A seasonal weighting factor was developed at the AASHO Road Test to account for the relative effects of axle loads applied at different times of the year (19). The factor, applied only to the flexible
pavements, ranged from 0 (no damaging effect) to 4.84 (very high damaging effect). The factors were low for summer months and were maximum during the spring when the detrimental effects of frost action and moisture were greatest.

Cumberledge, Cominsky, and Bhajandas (35) related pavement surface deflection measured with the Road Rater to the moisture content in the base and subgrade and to precipitation in Pennsylvania, Figure 2.14. They found that the deflection generally increases as the moisture content increased, except during the winter months when the deflection was constant due to the frozen condition of the pavement base and subgrade.

Pumping, a major problem related to rigid pavement performance, is associated with excessive moisture contents in the support materials beneath the slab. All failures in the rigid pavement sections at the AASHO Road Test were preceded by pumping of material from beneath the concrete slab (36). Both Yoder (37) and Cedergren (23) noted damage to pavement systems caused by pumping. These observations were especially prevalent where pavement overload and channelized traffic had occurred.

Thompson (25) indicated that the mechanisms whereby climate can influence pavement behavior and performance can be stated as follows:

1. Effect on the engineering properties of component materials.
2. Disintegration of materials caused by temperature and moisture changes (durability failure).
3. Temperature and moisture induced volume changes in component materials.

2.3.1 Effect on the Engineering Properties

The general response of a pavement system to traffic loading is controlled by the thicknesses of the various structural layers and the significant engineering properties of the paving materials and the subgrade soil. It is
apparent that if engineering property changes are affected by climate, pavement response will likewise be influenced.

Moisture content has a pronounced effect on the strength and deformation properties of soils. CBR-moisture content-density relations for the AASHO Road Test embankment soil, Figure 2.15, illustrate the typical effect (38). Extensive studies by Croney, Coleman and Black (39) in England have emphasized the development of relations between soil suction and soil stability, Figure 2.16 and 2.17. Sauer and Monismith (40) have studied the repeated load behavior of a glacial till at different suction values (moisture contents). Resilient moduli were substantially affected by soil suction, Figure 2.18.

Cumberledge, et al. (35) studied the seasonal relationships between subgrade moisture content and surface deflection. Figure 2.19 shows the results of one of the test sites studied. The deflections were measured by the Road Rater.

Barenberg and Tayabji (41) found that subgrade saturation was a major cause of pavement failure in a test track study of drainage of an open graded base material.

Water content, rather than temperature, is the major climatic factor influencing the strength and deformation properties of granular materials. Haynes and Yoder (42) found that the degree of saturation had a substantial effect on the repeated load deformation properties of the AASHO Road Test crushed stone and gravel materials, Figure 2.20.

A recent study by Thompson (43) emphasized the behavior of granular materials in pavement systems. As a result of the study it was concluded that granular materials at high levels of saturation become unstable under repeated loading. The importance of the subgrade soil condition (moisture content, strength) at the base course-subgrade interface was emphasized. Model study
results indicated that good interface conditions promoted improved repeated load behavior in the granular layer and the pavement system (43).

Substantiating data illustrating the detrimental effect of high moisture content on the repeated load behavior of granular materials have been presented by Snyder (44). Inference concerning the importance of moisture content relative to granular materials stability can be drawn from the many field studies of spring breakup which is typical in freezing zones.

Kesler (45) has stressed the importance of moisture and temperature conditions relative to concrete strength. Increased moisture content and higher temperatures will generally cause strength decreases. Davis and Troxell (46) have indicated that concrete compressive specimens tested in a wet condition display a higher modulus of elasticity than dry specimens.

2.3.2 Durability Failure

Typical paving materials (concrete, aggregates, asphaltic concrete, and stabilized soils) are vulnerable to deterioration due to the action of environmental factors (primarily temperature, moisture, and in the case of asphalt, sunlight). It is beyond the scope of this report to evaluate the durability properties of all paving materials. In some instances climate induced deterioration progresses to the extent that the materials are almost completely disintegrated. It is possible to have a pavement failure caused primarily by climatic factors and not by wheel loading.

Barenberg et al. (22) have identified most of the durability failures in flexible and rigid pavement systems. Thompson and Dempsey (47) have discussed the durability problems related to materials stabilized with lime, lime-fly ash, and cement.

Corrosion of reinforcing steel and dowel bars is a major source of durability failure in rigid pavement systems. This problem is especially
critical in areas where de-icing salts are used. Consideration must be
given to the durability characteristics of the component materials utilized
in the pavement system. These durability characteristics are a function of
the climatic conditions, geographical location, pavement materials, and
position in the pavement system.

2.3.3 Moisture and Temperature Induced Volume Changes

Volume changes associated with moisture and/or temperature changes in
the paving materials and the subgrade soils are major considerations in pave-
ment systems. Johnson, et al. (24) have summarized the volume changes that
occur in pavement systems as a result of frost effects. Lamb and Hanna (48)
have summarized the work that has been done concerning volume changes in ex-
pansive clays and clay shales.

Although it is not possible to discuss volume changes in detail, the
major effects are as follows.

1. Moisture and temperature induced volume decreases in concrete pave-
ment slabs produce cracks. Typically the slab is jointed in various ways
to alleviate the problem or continuously reinforced slabs are used to keep
the cracks tightly closed. Joints are a major source of roughness in a pave-
ment and are vulnerable to spalling. Pavement joints provide easy access for
surface moisture to the underlying pavement layers.

2. Moisture and thermal gradients in a concrete pavement are sufficient
to induce substantial stresses. Curling of the concrete slab is another
major consideration related to thermal gradients.

3. Drying shrinkage of some stabilized materials, particularly soil
cement, is sufficient to produce shrinkage cracking, thus breaking the con-
tinuity and integrity of the structural layer. Under most conditions, the
shrinkage cracks will reflect through an asphaltic concrete surface course layer.
4. Subgrade volume changes associated with frost action (moisture and temperature) or swelling soils (moisture) can produce substantial disruption of the pavement surface. In most cases, the heave is not uniform and pavement roughness is increased. There are situations where roughness increases to such an extent that the serviceability of the pavement system is substantially impaired.

Shober (49) has indicated that differential frost heaving in the vicinity of the pavement joints and cracks can cause frost tenting. He found that the conditions necessary for frost tenting are as follows:

1. Temperatures below freezing.
2. Salt in solution entering pavement crack.
3. Free water supply.
4. Saturation of pavement base and subgrade.

Carpenter, Lytton, and Epps (50) found that granular base course materials under asphalt concrete may experience volume changes as a result of freeze-thaw cycling. They indicated that the volume changes in the base can cause tensile cracks which can reflect through the asphalt concrete to the surface.

2.3.4 Summary

Climatic factors may effect the structural behavior and performance of flexible and rigid pavement systems. Numerous pavement distresses can occur as a result of changes in the engineering properties of the materials, disintegration of the materials, and volume changes.

2.4 Equilibrium in Soil-Water Systems

2.4.1 Soil Water Potential

Soil-water, like other bodies in nature, can contain energy in different forms and quantities. In science the two principal forms of energy are kinetic
energy which is a function of velocity and potential energy which is a function of position or internal condition of the system. Since water moves very slowly in soil the kinetic energy can generally be ignored in the study of soil-water systems. However, the potential energy is of primary importance in determining the state and the movement of water in soil.

The spontaneous and universal tendency of all matter in nature is to move from a point of high potential energy to a point of low potential energy until an equilibrium condition is reached. Soil-water systems obey the same universal pursuit of equilibrium.

A soil-water system is subjected to a number of force fields which causes its potential to differ from that of free water. The force fields commonly considered are gravitational potential, \( \phi_g \), pressure potential, \( \phi_p \), osmotic potential, \( \phi_o \), and gas potential, \( \phi_a \). The total potential, \( \phi_T \), of the soil-water system can be considered as the sum of the individual potentials as follows.

\[
\phi_T = \phi_g + \phi_p + \phi_o + \phi_a
\]  

(2-1)

The gravitational potential, \( \phi_g \), and pressure potential, \( \phi_p \), are the primary force fields in soil-water systems. The osmotic potential, \( \phi_o \), is dependent upon the presence of solutes in the soil-water system. The gas potential, \( \phi_a \), is dependent upon an external or internal gas pressure in the system. If the osmotic potential and gas potential are considered to have minor influence on the total potential then Equation (2-1) can be simplified as follows:

\[
\phi_T = \phi_g + \phi_p
\]  

(2-2)

Gravitational Potential, \( \phi_g \): It is generally known that every body on the surface of the earth is attracted by a gravitational force and that the weight of a body can be expressed as follows:
Weight = Mg \quad (2-3)

In Equation (2-3) \( M \) is the mass of the body and \( g \) is the acceleration of gravity. At a height \( Z \) above a randomly chosen reference level, the gravitational energy of water in soil can be stated as follows:

\[
E = MgZ = P_w gZV \quad (2-4)
\]

In Equation (2-4), \( P_w \) is the density of water, \( V \) is the volume of the mass, \( M \), and \( Z \) is the elevation above a reference level. From Equation (2-4) the gravitational potential energy, \( \phi_g \), can be expressed as follows:

\[
\phi_g = gZ \text{ (per unit mass } M) \quad (2-5)
\]

\[
\phi_g = P_w gZ \text{ (per unit volume } V) \quad (2-6)
\]

\[
\phi_g = Z \text{ (per unit weight } W) \quad (2-7)
\]

In Equation (2-7) \( \phi_g \) depends only on \( Z \) which is defined as the gravitational head in soil-water systems.

Pressure Potential, \( \phi_p \): The pressure potential is either negative for unsaturated soil-water systems or positive for saturated soil-water systems. Hillel (51) has indicated that the unified pressure potential concept allows for the consideration of the entire moisture profile in the field in terms of a single continuous potential extending from the saturated region to the unsaturated region, below and above the water table.

In general the positive pressure potential for a saturated system is well understood. The negative pressure potential has often been termed the capillary potential or more adequately the matric potential. This potential of soil-water results from the capillary and absorptive forces due to the soil matrix.

For unsaturated soil-water systems the capillary tube analogy is best used to discuss pressure potential. Soil can be assumed to be a porous media composed of different sizes of capillary tubes. In Figure 2.21 it is
observed that the air-water interfaces throughout the soil consists of menisci in which the curvature or radii indicate, similar to a capillary tube, the state of tension in the soil water. As the moisture content of the soil is reduced the water interfaces will recede into the smaller pores, the radii of curvature will decrease, and the moisture tension will increase.

In the capillary tube shown in Figure 2.22 the water above the water table will be in equilibrium when the upward component of the surface tension force is equal to the gravitational force acting on the suspended water. The height, \( h \), to which the water will rise in the capillary tube is related mainly to the surface tension, \( \sigma \), and radius, \( r \), of the meniscus by the following equation:

\[
h = \frac{2\sigma \cos \theta}{\rho_w g r}
\]  
(2-8)

In Figure 2.22 it is noted that atmospheric pressure exists at Points 1, 2, and 3. However, at point 4 just below the meniscus the pressure is less than atmospheric pressure by an amount equal to \( P_w gh \). By using the assumption made by Linford (52) that \( \theta \) is zero for water in soil, and that the curvature of the water in the soil matrix is similar to that in a capillary tube of the same size, Figure 2.21, then the pressure potential can be expressed from Equation (2-8) as follows per unit mass:

\[
\phi_p = -\frac{2\sigma}{r \rho_w} = gh
\]  
(2-9)

The negative sign is used in Equation (2-9) since the pressure potential in an unsaturated soil-water system is less than atmospheric pressure. Also, \( h \) would have a negative value in an unsaturated system.

From Equations (2-2), (2-5), and (2-9) the total potential, excluding the osmotic potential and gas potential, can be stated as follows per unit mass:

\[
\phi_T = gZ + gh
\]  
(2-10)
On a unit weight basis which is normally used in soil-water studies Equation (2-10) can be shown in the following form:

\[ H = h + Z \] (2-11)

In Equation (2-11), \( H \) is the total soil water head, \( Z \) is the gravitational head, and \( h \) is the pressure head. The pressure head or suction head is a negative head in unsaturated soil-water systems and a positive head for saturated soil-water systems.

In summary, the criterion for equilibrium in soil-water systems is that the total water potential be equal throughout the system. In order to facilitate the analysis of particular systems the total water potential is partitioned into various components which can be measured. Typically the gravitational potential is determined by use of a measuring tape, the pressure potential by a piezometer for saturated systems and a tensiometer for unsaturated systems, and the gas potential by a pressure gage.

2.4.2 Units for Soil-Water Potential

Mass is not the only property used to characterize the quantity of a body. Commonly in thermodynamics, energies are expressed on a per mole basis. Conversion between the per mass and per mole basis merely involves multiplication or division by the mass per mole of the substance considered.

Two other bases for expressing the quantity of a body are frequently used in soil water work, but almost never in thermodynamics. These involve expressing energies on a per unit weight and on a per unit volume basis. All three of the bases used in soil-water work, energy/mass, energy/weight, and energy/volume, are called "potentials."

In a gravitational field of constant intensity there is a simple relation between the mass and the weight of a body, \( \text{weight} = g \times \text{mass} \), so the conversion between potentials expressed on a per unit mass basis and a per unit weight
basis just involves multiplying or dividing by the force of gravity.

In general there is not a constant relationship between the mass and the volume of a body. For gases this relationship depends strongly on the pressure and the temperature of the gas. Hence in general it is not wise to express the energy of a body on a per unit volume basis. A "potential" defined as energy/volume will change when either the energy or the volume changes.

However, in the study of soil water relations the material considered is liquid water. The relation between its mass and volume is its density which does not change much when pressure and temperature vary over the range of values normally found in soil-water systems.

The normal methods of expressing potential in soil-water systems are shown in Table 2.1 for the various measurement systems. Relationships for the measurement systems are shown in Table 2.2. For potentials expressed on a per unit weight basis or on a per unit volume basis the dimensions are those of a length (cm, meter or ft) or of a pressure (dyne/cm², Newton/meter² or lb/ft²), respectively. Equations for converting between the three forms of potential are stated as follows:

\[
\frac{\text{energy}}{\text{mass}} = g \frac{\text{energy}}{\text{weight}} \quad (2-12)
\]

\[
\frac{\text{energy}}{\text{volume}} = P \frac{\text{energy}}{\text{weight}} \quad (2-13)
\]

\[
\frac{\text{energy}}{\text{volume}} = g P \frac{\text{energy}}{\text{weight}} \quad (2-14)
\]

When analyzing equilibrium in soil-water systems it is convenient to consistently use one of the three methods for expressing potential rather than changing between them. Of the three methods, potentials expressed as energy per unit weight appear to be utilized most in the literature.

2.4.3 Soil-Water Characteristic Curve

Observations on unsaturated soil columns in equilibrium with free water
have shown that the moisture content decreases with height above the free water. This is equivalent to stating that moisture content decreases as the pressure potential becomes more negative. This phenomenon is commonly expressed in the form of a "soil-water characteristic curve." Typical curves for seven soil types are shown in Figure 2.23 (53).

The relationship expressed in a soil-water characteristic curve is a soil property which is of fundamental importance in the analysis of a soil's equilibrium and flow behavior. Physically the curve tells (at any given moisture content) how much energy (per unit quantity of water removed) is required to remove a small quantity of water from the soil. It indicates how tightly water is held in the soil. Hillel (51), Taylor and Ashcraft (53), Kirkham and Powers (54), and Rose (55) have presented detailed explanations of how water is held in soil. Childs (56) has considered the mechanisms of water held in both swelling and non-swelling soils in great detail.

Croney, Coleman, and Bridge (57) have described the methods used to determine the soil-water characteristic curve. Generally these methods consist of the tensiometer method, the direct suction methods, the pressure plate method, and the centrifuge method. Usually no single method can be used to cover the entire moisture tension range, so several measurement methods may be used in actual practice.

Figure 2.24 shows a simple type of tensiometer system which can be used for the low moisture tension range (< 1 bar). The apparatus shown in Figure 2.24 consists of a porous plate with its pores filled with water. The chamber beneath the porous plate is filled with water and connected to a flexible tube which is also filled with water. The negative head is equal to the distance between the soil sample and the outflow end of the flexible tube. The soil-water characteristic curve is determined from the relationship between the water content of the soil sample and the magnitude of the negative pressure head of the water.
Hysteresis effects such as that shown in Figure 2.25 will often occur between drying and wetting soil water characteristic curves. The hysteresis for the drying and wetting conditions arises from the degree of control the smaller pores have on the water held in the larger pores.

Hillel (51) has indicated that the soil water retention in the low tension range (0-1 bar) is strongly influenced by soil structure and pore-size distribution. Therefore, measurements made with disturbed samples cannot be expected to represent field conditions. Hillel (51) recommended that undisturbed field cores be used for the low tension measurements. However, Hillel (51) did indicate that soil-water retention in the high tension range is due primarily to absorption and is therefore correlated with the specific surface of the soil rather than with its structure.

2.5 Flow in Soil-Water Systems

2.5.1 Darcy's Law

Analysis of soil water flow systems is a highly empirical science. It is based almost entirely upon the seeming universality of an empirically derived statement, known as Darcy's Law, which has the following general differential form.

\[ q = -K \frac{\partial H}{\partial x} \]  (2-15)

Darcy's Law states that the rate of flow of water, \( q \), in any direction in a porous medium is proportional to the rate of change of the hydraulic potential, \( H \). The hydraulic conductivity, \( K \), is the proportionality constant for flow.

In terms of the \( x \), \( y \), and \( z \) directions Darcy's Law is expressed as follows:

\[ q_x = -K_x \frac{\partial H}{\partial x} \]  (2-16)

\[ q_y = -K_y \frac{\partial H}{\partial y} \]  (2-17)

\[ q_z = -K_z \frac{\partial H}{\partial z} \]  (2-18)
The hydraulic conductivities $K_x$, $K_y$, and $K_z$ in Equations (2-16), (2-17), and (2-18) respectively may or may not be equal. If they are all equal, the porous medium is isotropic. If not, the porous medium is anisotropic. Childs (57) has indicated that Equations (2-16), (2-17), and (2-18), are valid for anisotropic porous media only if $x$, $y$, and $z$ are the principle axes of the medium with respect to hydraulic conductivity.

Darcy's Law, though originally conceived for saturated flow only, was extended by Richards (58) to unsaturated flow, with the provision that the conductivity is a function of the matric suction head. The unsaturated flow equation is expressed as follows:

$$q = -K(e) \nabla H$$  \hspace{1cm} (2.19)

In Equation (2-19) $K(e)$ is a function of unsaturated water content and $\nabla H$ is the hydraulic head which is a function of the suction head and gravitational head [see Equation (2-11)].

Equation (2-19) has been found to be valid for a range of flow systems including unsaturated flow by Kirkham and Powers (54) and Childs and Collis-George (59) and non-steady flow by Rogers and Klute (60).

When Darcy's Law is applied to unsaturated flow situations, the values of the hydraulic conductivity are found to be highly variable with moisture content. Hydraulic conductivity is largest when the soil is water-saturated, decreasing as the moisture content decreases. Typically, hydraulic conductivity as a function of moisture content can also be expressed as a function of matric potential since moisture content is a function of matric potential. Bouma (61) has developed hydraulic conductivity-matric potential curves for some major soil horizons, Figure 2.26.

It should be noted that hydraulic conductivity data like those presented in Figure 2.26 are highly soil type dependent, and even site dependent. Other soils of the same texture classes may deviate considerably from these curves.
because of differences in soil structure. This problem applies even more to hydraulic conductivity data than to moisture characteristic data like those presented in Figure 2.23.

2.5.2 Transient Flow

The laws necessary for consideration of transient flow systems are the extension of Darcy's Law to unsteady flow systems and the principle of the conservation of matter. The equation of continuity which is a statement of the law of conservation of matter can be written as follows:

\[
\frac{\partial \phi}{\partial t} = \mathbf{\nabla} \cdot (K(\theta)\mathbf{\nabla}H) \tag{2-20}
\]

In Equation (2-20), \( \frac{\partial \phi}{\partial t} \) is the change of moisture content with time in the soil-water system. By combining Equations (2-19) and (2-20) the following general transient flow equation is obtained:

\[
\frac{\partial \phi}{\partial t} = \mathbf{\nabla} \cdot (K(\theta)\mathbf{\nabla}H) \tag{2-21}
\]

Since the total head is equal to the sum of the pressure head and gravitational head (Equation 2-11), Equation (2-21) can be written as follows for one-dimensional transient moisture flow in the vertical direction:

\[
\frac{\partial \phi}{\partial t} = \frac{1}{2} \left( K(\theta) \frac{\partial h}{\partial z} + \frac{\partial \theta}{\partial z} \right) \tag{2-22}
\]

or

\[
\frac{\partial \phi}{\partial t} = \frac{1}{2} \left( U(\theta) \frac{\partial \theta}{\partial z} \right) + \frac{\partial \theta}{\partial z} \tag{2-23}
\]

In Equation (2-23), \( U(\theta) \) is called the soil-water diffusivity and it is equal to \( K(\theta) \frac{\partial h}{\partial z} \).

In summary, the analysis of transient flow includes the extension of the equilibrium relation between soil-moisture content and matric potential to flow situations, Darcy's Law, and the law of conservation of matter.

2.5.3 Simultaneous Flow

The flow equations which have been considered previously do not include the influence of solutes or temperature gradients on moisture movement. Hillel
(51) has discussed simultaneous water and solute movement in soils and this area will not be considered further. However, in airport pavements the simultaneous movement of heat and water is a common occurrence and very important to the development of design methodologies to resist the influence of climatic effects.

The fact that temperature gradients can induce water movement in soils has been known for the last 50 years (62). Studies on the relative importance and interaction of thermal and suction gradients in transporting soil moisture have been carried out by Hutchinson, Dixon, and Denbigh (63), Philip and de Vries (64), Taylor and Cary (65, 66), Cassel (67), Cary (68), Hoekstra (69), and Jumikis (70).

At present time there are two classical theories for explaining moisture movement under simultaneous moisture and thermal gradients. These theories were developed by Philip and de Vries (64) and by Taylor and Cary (65, 66).

Taylor and Cary (65) applied the theory of irreversible thermodynamics to the study of water, heat, and salts through soil systems. Using the theory of irreversible thermodynamics, Taylor and Cary (65) developed a linear flow equation for each component of the soil system which has the following general form:

\[ J_i = \sum_{k=1}^{n} L_{ik} X_k \]  \hspace{1cm} (2-24)

In Equation (2-24) \( J_i \) represents the mutually interacting fluxes resulting from forces such as diffusion, temperature, and pressure, \( X_k \). The term \( L_{ik} \) represents the transmission coefficients of the various fluxes such as the diffusion coefficient, hydraulic conductivity, and thermal conductivity and \( n \) is the number of driving forces. For the movement of moisture under a combined moisture and temperature gradient Equation (2-24) becomes:

\[ J_u = L_{wu} X_u + L_{ww} X_w \]  \hspace{1cm} (2-25)
In Equation (2-25) $L_{\text{wu}}X_u$ is the flux caused by the temperature gradient and $L_{\text{ww}}X_w$ is the flux caused by the moisture gradient.

From a mechanistic approach, Philip and de Vries (64) developed the following equation for water movement under a combined moisture and temperature gradient:

$$Q = D_\Theta \nabla \Theta + D_T \nabla T + K(\Theta)$$  \hspace{1cm} (2-26)

In Equation (2-26) $Q$ is the net water flux, $D_\Theta$ is the isothermal moisture diffusivity, $\nabla \Theta$ is the moisture content gradient, $D_T$ is the thermal moisture diffusivity, and $\nabla T$ is the temperature gradient. The terms $D_\Theta$ and $D_T$ are made up of two components each, one for vapor flow and one for liquid flow. The term $K(\Theta)$ is the gravity term.

In 1964, Dirksen (71) used the Philip and de Vries model to predict moisture movements in frozen soils. In his study, Dirksen (71) made the assumption that the soil water diffusivity values could be expressed as exponential functions.

In 1968, Cassel (67) compared his experimental results to the predictions of the models of Philip and de Vries and Taylor and Cary. His results were as follows:

1. The Taylor and Cary theory of irreversible thermodynamics did not acceptably predict the observed net soil-water flux for any time intervals for soil columns with an initial water content of 0.101 cm$^3$/cm$^3$ or 0.077 cm$^3$/cm$^3$.

2. The Philip and de Vries theory acceptably predicted the observed net soil-water flux in response to an imposed temperature gradient of 0.493 C/cm for an initial water content of 0.077 cm$^3$/cm$^3$.

3. The apparent values for $D_T$ and $D_\Theta$ were 1.2 and 3.0 times greater respectively than the values presented by Philip and de Vries.
In general, the temperature of the soil influences water transport through its effect on the forces that cause the water to move, and through its effect on the conductivities and diffusivities in the various flux equations. In any given application of flow theory, the significance of the water transport due to nonisothermal conditions must be assessed. Water flow in response to a temperature gradient is usually in the direction of decreasing temperature. In porous media with a gas phase, a temperature gradient produces an associated vapor pressure gradient and a surface tension gradient. There is then a component of the vapor flux that can be related to the applied temperature gradient, and a component of the liquid flux which is associated with the temperature gradient.

2.6 Methods of Predicting Moisture Conditions in Pavement Systems

2.6.1 General

The engineering problems associated with the behavior of pavement systems in response to moisture changes have been widely studied with reference to the mechanisms of moisture movement and to the consequences of such moisture changes. The chief task is that of quantitatively and qualitatively predicting moisture movement and moisture equilibria in pavement systems at any particular place, depth, and time.

Climate is the dominant factor influencing the space-moisture conditions in pavement systems. Other variables are permeability through the pavement profile, the type of cover and surrounding vegetation, and the local topography. Surface runoff from the pavement, drainage conditions, water table location, and pavement edge conditions are also important factors.

Moisture movement and moisture equilibria in soils have been major concerns in the field of soil science and agriculture for some time (72, 73, 74, 75, 76). It has only been in recent years that a number of investigators
(14, 16, 77, 78) have attempted to analyze moisture conditions in pavement systems. The OECD (1) has indicated that the general methods for predicting moisture movement and moisture equilibria in pavement systems can be placed into two categories as follows:

1. Empirical methods based on experience and field studies.
2. Theoretical methods based on the equations governing moisture conditions in porous materials.

2.6.2 Empirical Methods

Because of the complexity of the thermodynamics governing moisture movement in pavement systems, a substantial number of methods for predicting moisture movement and moisture equilibria of in-service pavements have been based on accumulated experience and empirical relationships reflecting this experience.

The Thornthwaite moisture index, which relates subgrade moisture conditions to climatic indices, has been a popular means of empirically estimating moisture conditions in pavement subgrade soils (79). Thornthwaite (79) has pointed out the precipitation alone does not indicate whether a climate is moist or dry. It must also be known whether precipitation is greater or less than the water needed for evaporation and transpiration. Where precipitation exceeds water need, the climate is moist and where it is less than water need, the climate is dry.

Thornthwaite (79) termed the combined evaporation from the soil surface and transpiration from plants as "evapotranspiration." He called the amount of water that would evaporate and transpire if it were available "potential evapotranspiration." He indicated that evapotranspiration and precipitation are equally important climatic factors.

Potential evapotranspiration must be determined experimentally. Since the determination of potential evapotranspiration is difficult, Thornthwaite
(79) established a relationship between potential evapotranspiration and other potential weather elements as follows:

\[ E = 1.6 \left( \frac{10T}{T} \right)^a \]  

(2-27)

where

\[ E = \text{potential evapotranspiration (in.)} \]
\[ T = \text{the mean monthly air temperature (C)} \]
\[ I = \text{annual total of (T/5)} \]
\[ a = 0.000000675 I^3 - 0.0000771 I^2 + 0.017921 + 0.49239 \]

Equation (2-27) is based on a month of 30 days and 12 hours of daylight. A daylight correction factor can be applied depending on the latitude of the area investigated.

The overall availability of moisture in soil during any given year can be assessed by using the Thornthwaite moisture index which is as follows:

\[ \text{Moisture Index} = \frac{100D - 60d}{E} \]  

(2-28)

where

\[ D = \text{the subsurface and surface drainage or moisture surplus (in.)} \]
\[ d = \text{the moisture deficit in the soil (in.)} \]

Monthly or daily values of potential evapotranspiration and precipitation can be used to determine the amount of moisture stored in the soil and the amount of surplus or deficit moisture available.

The Thornthwaite moisture index is positive for subgrades with high moisture content and negative for subgrades with low moisture contents.

Several investigators (78, 80) have successfully used the Thornthwaite Moisture Index for assessing soil moisture and ground water levels beneath pavement surfaces. It is apparent that the quantitative procedure for separating from precipitation the amount of water used by evaporation and
transpiration is an important development.

Haas (81) has used a natural drainage index to describe the natural moisture condition of the soil profile. The value of the drainage index is influenced by both the texture or relative permeability of the soil profile and the landscape position and slope, which governs both surface and subsurface drainage. In the system Haas (81) used an arbitrarily assigned value of +1 for well drained soils. An organic or bog soil which is very poorly drained is rated as +10 and a soil which is very excessively drained is rated -10. Soils with drainage characteristics between the two extremes are assigned appropriate numbers to indicate their relative position in the drainage sequence.

In conclusion, Haas (81) found that there was a very good correlation between the natural drainage index of a soil series and recognized engineering classifications, such as the AASHO classification, the Corps of Engineers frost-susceptibility ratings, and pumping criteria. He also found that there was a very good correlation between the drainage index and the actual performance of pavement systems. It is evident that the natural drainage index concept may offer an expedient means of organizing many soil series into workable moisture classifications for engineering purposes.

In application of the AASHO Road Test results to the design of flexible pavement systems, Liddle (82) has included a "regional factor" which includes the adverse effects of climate. The regional factor has a value between zero and five and is estimated by analyzing the duration of certain typical conditions such as load applications, water table elevation, topography, and subgrade strength. Much of the data used to determine the regional factor are estimated on the bases of judgment.

The OECD (1) has summarized some of the relationships between water content and soil characteristics such as the plastic limit, liquid limit,
water content, and particle size.

Swanberg and Hansen (83) found that the water contents of highway sub-
grades in Minnesota could be estimated in terms of the plastic limit, PL, as
follows:

\[ w = 1.16 \ PL - 7.4 \]  
\( (2-29) \)

The subgrades were mainly clayey silt soils with plastic limits from 15 to
30 percent. The density was 90 to 105 percent of the maximum density by the
Modified Proctor Test, AASHO T99-38. The water contents measured in the
spring were about 0.8 percent higher than those measured in the summer.

The American Navy (84) investigated the sandy and clay subgrades of 70
airports where the ground water table was greater than 60 cm below the sur-
fice. They presented the following equation:

\[ w \leq PL + 2 \]  
\( (2-30) \)

The major conclusion was that the water content of the subgrade exceeded
the plastic limit by about 2 percent.

Kersten (21) investigated the subgrade water contents of airport pave-
ments in seven states with damp to desert climates. The water contents were
observed in the top 30 cm of the subgrade soil. Kersten (21) found that for
the damp regions the water content for sand and clay soils was between 80
percent and 120 percent of the plastic limit. He presented the following
relationship:

\[ 0.8 \ PL < w < 1.2 \ PL \]  
\( (2-31) \)

The Bridges and Highways Central Laboratory of France found that high-
way subgrades in many parts of France have water contents which also lie
within the limits of Equation (2.31). Wooltorton (85) developed the following
equation based on a large amount of published experimental data:

\[ w = 1.17 \ PL - 4 \]  
\( (2-32) \)
Equation (2-32) was developed for plastic subgrades with a plasticity index between 6 percent and 28 percent.

The Corps of Engineers (8) studied subgrades of flexible airport pavements in New Mexico, Texas, Tennessee, Mississippi, and Oklahoma. They made the following general observations based on the study:

1. The water contents observed generally increased with liquid limit.
2. No correlation between water contents of the soil and amount of annual rainfall was observed.
3. The water contents were often near the plastic limit and rarely exceeded the liquid limit.
4. The degree of saturation of plastic soils exceeds 85 percent most of the time regardless of the annual rainfall.

The OECD (1) has indicated that many of the empirical formulas which relate the water contents observed in-situ in highway and airport subgrades to certain characteristics of the soils, cannot really be described as methods of prediction. They indicate that the results of the various investigations are always scattered around regression lines which represent the average of the observations, with a standard deviation representing several percentages of water content. Figure 2.27 shows that the different investigations have led to different regressions and that a general treatment of all the data collected would give, in regards to the relation between water content and plastic limit, a zone extending approximately 4 percent on either side of the line of equality. This is a fairly wide range in which the variation of the mechanical properties of cohesive soils may be quite large. For example, the CBR could vary by a factor higher than 100 percent.

The OECD (1) has listed the following causes for the considerable dispersion of the empirical formulas:
1. The water contents observed have not been correlated with more fundamental characteristics of the nature of the soils such as the specific surface area and the mineralogical composition.

2. The compaction of the soil has not been measured, with the exception of those investigations which give some consideration to the degree of saturation. It is obvious that the quantity of water which a soil can absorb in given circumstances varies with compaction.

3. The degree of waterproofing of pavements has not been measured or evaluated, in general the type of pavement has not even been mentioned.

4. The edge effect seems to have been disregarded in most of the studies.

5. The absence or presence of a ground water table and its depth generally does not appear to have been taken into consideration. This is also the case of the water content of adjacent soil masses.

6. The effects of climatic factors have not been disclosed.

The conclusion that the empirical formulas will generally give rather poor solutions to the problem of forecasting water contents should not mean that considerable local interest may not be attached to the investigations. It is evident that limited engineering design decisions can be made based on the hydrological behavior of soils in the regions where the investigations were performed.

2.6.3 Theoretical Methods

In studies of moisture movement and moisture equilibria, it is evident
that experience and empirical relationships will often be inaccurate and impractical because of the rapidly changing climatic conditions which generally occur and influence pavement systems. It becomes apparent that formal mathematical procedures based on thermodynamic principles which incorporate the necessary daily environmental boundary conditions of a given geographical area are needed.

A rational method based on the thermodynamic theory of equilibrium distribution of water in a porous body has been developed by researchers of the British Road Research Laboratory (86, 87, 88, 89). The procedure for using the rational method has been described in a report prepared by the OECD (1). The method is based on three principles as follows:

1. The trend in pore water pressure, under certain conditions at a given level of the subgrade, is towards an equilibrium value depending solely on the height above the ground water level.

2. The existence of a relation between the pore water pressure in the soil at a given level and the suction of the soil.

3. The existence of a relation between the suction and the water content of the soil.

The conditions for equilibrium to be reached depend upon the following:

1. The temperature of the subgrade is constant, uniform, and above freezing.

2. The subgrade does not receive moisture by infiltration through the highway pavement or by migration from adjacent soil masses with a higher pore water pressure, nor does it give up moisture by evaporation or migration to adjacent soil masses having a lower pore water pressure.
In the rational method the pressure of the pore water at any given level must tend towards an equilibrium which cancels out the algebraic sum of the various water potentials. The British Road Research Laboratory (86, 87, 88, 89) expressed the equilibrium condition as follows:

\[ u = -Z \]  \hspace{1cm} (2-33)

In Equation (2-33) \( u \) is the relative porewater pressure which is negative above the water table and \( Z \) is the height above the ground-water table. Graphically when the same scale is used for pressures \( u \) and heights \( Z \), the function \( u = f(Z) \) is a straight line with slope of \( 45^\circ \) regardless of the nature and dry density of the various soil layers making up the mass in question. In principle, estimating the equilibrium pressure profile of a highway subgrade is dependent upon estimating the position of the ground water level after building the highway.

By definition soil moisture tension or suction is the value of the pore water pressure of an undisturbed sample which, because of its removal, has been relieved of all external stress. The suction of the soil after removal can be represented by the following equation:

\[ s = u - \alpha P \]  \hspace{1cm} (2-34)

In Equation (2-34) the suction, \( s \), and the pore water pressure, \( u \), are negative. However, the magnitude of \( s \) is greater than \( u \) because prior to removal the soil sample was subjected to a vertical pressure, \( P \), caused by the overburden weight of the soil, pavement, and load. When the sample is taken, the stress release causes a volume increase and a corresponding decrease in the pore water pressure. This fact is usually accounted for by the compressibility factor, \( \alpha \), in the expression.

The distribution of the in-situ overload stress between the water pressure and the soil intergranular pressure has been studied by Bishop (90) and
by Skempton (91). This shows that for non-saturated, non-cohesive soils, the factor $\alpha$ is generally near zero, while for cohesive soils $\alpha$ increases with the degree of saturation and reaches the value 1 when saturation is complete.

According to the OECD (1), the value of $\alpha$ for a cohesive soil can be obtained from the following approximate formula:

$$\alpha = 0.03 \ PI \hspace{1cm} (2-35)$$

In Equation (2-35) PI is the value of the plasticity index.

The application of the rational method developed by the British Road Research Laboratory (86, 87, 88, 89) and as described by the OECD (1) for predicting the equilibrium water content in subgrade soil is shown in Figure 2.28. The procedure for developing Figure 2.28 is as follows:

1. Calculate the values of the overburden pressure, $\mathbf{P}$, and the product of $\Delta \mathbf{P}$ at the desired depths.

2. For the desired depths determine the equilibrium pore water pressure by Equation (2-33) and the equilibrium suction by Equation (2-34).

3. The equilibrium water contents corresponding to the equilibrium suction determined in 2 above are obtained from the soil water characteristic curve $w_{eq} = f(s)$

4. The equilibrium water contents for various points in the profile are determined graphically from the $w_{eq} = f(Z)$ relationship.

The OECD (1) has indicated that the rational method is a valuable tool for predicting water content of soils even though it has limitations. The fact that it is necessary to determine the applicability of the method by examining the ground water level, the permeability of the pavement structure,
and the climatic factors will provide much useful information. Even if conditions for application of the rational method are unfulfilled, the procedure will show how the water profile is different from the equilibrium profile and provide the pavement designer with guidance for choosing measures to prevent adverse water conditions.

Several investigators (64, 92, 93, 94, 95, 96) have proposed mathematical formulas based on thermodynamic principles for predicting moisture movement caused by nonisothermal and isothermal conditions. The literature indicates that the important liquid and vapor diffusivity parameters for defining the potential causing moisture movement in the formulas can be expressed quantitatively in terms of soil properties and soil suction.

Klute (92) and Selim (94) have developed reasonable models for predicting moisture movement in soils subjected to isothermal conditions. These models are based on finite-difference solutions to the differential equations for one- and two-dimensional moisture movement.

Selim (94) developed a two-dimensional transient and steady state water flow computer model for determining water content in soils. Numerical methods were used to develop the computer model. Selim (94) solved the transient flow equation by using the alternating-direction implicit method. The steady state flow equation was solved by using the successive overrelaxation iterative method. The two-dimensional flow medium considered was a homogeneous soil with an impervious barrier (or a groundwater table) at some depth from the soil surface and equally spaced trenches (or irrigation furrows and ridges) to which water was supplied.

Numerical results of the water content distributions, the water flux, and the rate of water flow were obtained for Ida silt loam and Webster clay loam soils. Several dimensions of the flow medium were used.
In the transient state, Figure 2.29, the water content distributions show that water moves from the water source at the bottom of the trench in two dimensions. The maximum movement is downward due to the gravitational force. Figure 2.29 also shows that the rate of water flow in the flow medium decreases with time. In the presence of a groundwater table source, Figure 2.30, the water movement from the groundwater table is strictly one-dimensional in the upward direction until the wetting fronts associated with the two sources join together.

In the steady state, Figure 2.31, equal water lines of high water contents extend to the impervious barrier, and high water content gradients occur at the soil surface boundary. In the presence of a groundwater table, Figure 2.32, steady state water fluxes are negative. These fluxes indicate that in the steady state a groundwater table is a water sink rather than a source of water.

Results of the transient and the steady state flow show that as time increases the transient flow approaches that of the steady state.

Selim (94) was not able to compare his theoretical results with field data. However, he indicated that the theoretical and experimental flow patterns would agree.

Elzeftawy (97) compared measured water contents from both the laboratory and field with water contents predicted by the model developed by Selim (94). In Figures 2.33 laboratory water contents are compared with predicted water contents after various time periods in a Lakeland fine sand for a surface infiltration rate of 2 cm/hr. The initial water content of the soil was 0.109 cm$^3$/cm$^3$.

Figure 2.34 shows a comparison between predicted and measured water contents in the field. The surface infiltration rate of the field test was 2.03 cm/hr. It is evident from the work by Elzeftawy (97) that the model
developed by Selim (94) accurately predicted isothermal field moisture contents with time.

Selim (94) concluded that the alternating-direction implicit method for solving transient two-dimensional water flow equations is convergent and stable and gives results which appear reasonable. The technique provides for the prediction of soil water content in a flow medium at any location and time if the soil properties are known. The technique is flexible and can incorporate hysteresis, soil nonhomogeneity, changes in initial and boundary conditions, and geometrical dimensions.

Richards (93) has been quite successful in using computer methods for predicting the time-space moisture conditions in pavement systems subjected to isothermal conditions. Richards (93) used a two dimensional model, but he analyzed only the isothermal condition. Richards (93) also implemented a field study to check his model. Richards (93) concluded that while his program provided a valuable tool for analyzing subgrade moisture conditions, there was inadequate field data for conclusive results to be made about the accuracy of the model.

In 1970, Lytton and Kher (95) developed a model to predict moisture movement in expansive clay subgrades. They analyzed only the isothermal case, but they had both one and two dimensional programs, and used a field study to validate the model.

Figure 2.3 shows a typical comparison between field moisture content and predicted moisture for various depths below the surface of a field test site. The field data was obtained from a test site near Laramie, Wyoming. Based on Figure 2.35, it would appear that Lytton and Kher (95) were successful in accurately predicting moisture in an expansive clay subgrade soil. In demonstrating the broad scope of their program, Lytton and Kher (95) presented
predictions of moisture contents around a pipe casing, beneath a house, in a stratified clay layer subjected to ponding, and in a concrete girder. Lytton and Kher (95) concluded that excellent results could be achieved by use of their program provided the input data was of high quality.

2.6.4 Summary

The mechanisms responsible for moisture fluctuations in pavement systems are very complex. There are substantial interactions between the pavement moisture conditions and the extrinsic climatic factors and intrinsic pavement factors.

Based on the extensive field, laboratory, and theoretical studies which have been discussed, it would appear that some progress has been made to develop quantitative methods for predicting moisture conditions in pavement systems as a function of time and space. As pointed out by the OECD (1), the development of methodologies for controlling moisture conditions in pavement systems will rely on the ability to accurately predict the moisture regime as a function of the pavement and climate parameters.
3.1 General

The laboratory study was conducted to validate the moisture model for controlled conditions. By using a gamma-ray transmission method for nondestructive measurements of water content and a tensiometer-pressure transducer arrangement for measuring soil water suction, the unsaturated hydraulic conductivity, soil water diffusivity, and soil water content-suction characteristic functions were evaluated for an AASHO A-4 subgrade soil in a compacted soil column in the laboratory under isothermal condition. A water table was established at the bottom of soil column and the transport of water through subgrade soil was studied. The data from the laboratory study was utilized in the moisture model to predict the position and distribution of water in subgrade soils.

3.2 Theory

In 1822, Fourier (98) presented a very complete mathematical theory of the transport of heat in conducting materials based on the law that the rate of conduction of heat is proportional to the temperature gradient. In 1827, Ohm (99) enunciated a law to the effect that the rate of transport of electricity (i.e., the strength of an electric current) in a conductor of electricity is proportional to the difference of electrical potential between its ends, and from this it is a short step to the law for conductors of different length but of the same material and cross-section, i.e., that the electric current is proportional to the electric potential gradient. In 1845, the Navier equations describing the flow of viscous fluids in terms of the distribution of hydraulic potential
were presented by Stokes (100). Where the boundaries of the moving fluid are simple, as in the case of flow through a tube of uniform radius, these equations permit the derivation of flow rates in terms of the dimensions of the conductor and the potential difference between the ends. Poiseville (101) experimentally derived the equation of flow of fluid through a tube. It can be written in the form,

\[
Q/t = (\Delta \phi/L) \left( \frac{\pi}{8\eta} \right) R^4 \rho \tag{3-1}
\]

where, \( Q \) is the volume passing in time, \( t \), \( L \) is the length of the tube between the ends of which potential difference is \( \Delta \phi \), (i.e., \( \Delta \phi/L \) is the potential gradient), \( \eta \) is the viscosity of fluid and \( \rho \) is its density, and \( R \) is the radius of the tube.

The configuration of the pore space in a porous material, such as soil, is far too complicated and unspecifiable in quantitative terms to permit the rate of flow of fluid to be calculated by an application of the Stokes-Navier equations, although the general form of the law can be demonstrated. However, Darcy (102) was able to formulate the law as a result of experiments on the infiltration of water through filter beds of sands. In his classic study on isothermal flow of water through saturated sand, he observed flow velocity to be directly proportional but in opposite direction to the hydraulic gradient imposed across the bed of sand. Darcy's law has been widely accepted as the basis for describing flow of liquids in saturated and unsaturated porous material. In 1950, Childs and Collis-George (59) introduced the soil-water diffusivity concept as a means to describe flow in water-unsaturated soils. Their theory for unsaturated flow of water assumes Darcy's law can be written as a
diffusion-type water flow equation in homogeneous soils where gradients of water content rather than gradients of total potential are expressed as:

\[ q = -D(\theta)\nabla \theta + K(\theta) \quad (3-2) \]

where:
- \( q \) = the water flux (cm/hr),
- \( \theta \) = the soil water content on volume basis (cm\(^3\)/cm\(^3\)),
- \( K(\theta) \) = the soil hydraulic conductivity (cm/hr),
- \( C(\theta) = \frac{\partial \theta}{\partial h} \), the specific soil water storage capacity (cm\(^{-1}\)) where \( h \) is the soil water suction (cm),
- \( D(\theta) = K(\theta)/C(\theta) \), the soil water diffusivity (cm\(^2\)/hr),

and \( \nabla \theta \) = the water content gradient (cm\(^{-1}\)).

Both \( D(\theta) \) and \( K(\theta) \) are functions of soil water content, \( \theta \). Equation (3-2) can also be written in the form:

\[ q = -K(\theta)\nabla H \quad (3-3) \]

where \( H \) is the hydraulic head (cm). The advantage of Equation (3-3) over (3-2) is that it is a general form which can be used for unsaturated heterogeneous soils.

Equation (3-2) resembles Fick's Law of Diffusion with a concentration dependent diffusivity, except for the \( K(\theta) \) term which arises from the gravitation component of total hydraulic head. Combining Equation (3-2) with the equation of continuity (i.e., conservation of mass) yields a "diffusion-type" equation for flow in porous media under isothermal conditions (Kirkham and Powers (59)), which can be written in the form:

\[ \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\{D(\theta) \frac{\partial \theta}{\partial z}\} - \alpha K(\theta)/az \quad (3-4) \]
where \( t \) is the time, and \( z \) is the vertical space coordinate. The term \( \frac{\partial K(\theta)}{\partial z} \) in Equation (3-4) is normally referred to as the gravitational component.

The validity of Equation (3-4) in describing the flow of water in unsaturated soils has been demonstrated by several researchers (103, 104, 105). For a review and detailed derivation of the laws governing soil water movement, the reader may refer to an extensive review article by Gardner (106) in which over 300 references were cited.

3.3 Materials and Methods

3.3.1 Laboratory Equipment

In order to be able to experimentally investigate most infiltration-redistribution and drainage problems for the range of soils and subgrade whose pore geometry remains constant during water content changes, it is necessary to use soil columns approximately 4 ft to 5 ft in length. In the present equipment, provisions have been made in the design both of the column and traversing mechanism for a column length of approximately 5.9 ft.

As will be detailed later, the nondestructive measurement of water content in this equipment utilizes the principle of gamma-ray adsorption which necessitates, external to the soil column a radioactive source on one side and a scintillation counter on the other. This assembly of source and detector must then be positioned at the elevation where the measurement of water content change is desired.

The general design requirements of the traversing mechanism then emerge as follows:
1. The mechanism must be sufficiently strong to move the source and detector, together with the heavy shielding involved, up and down the column with a minimum of vibration and with accurate horizontal positioning of the collimation slit relative to the column center line,

2. The range of this movement must be equal to the height of the soil column,

3. The source and detector assembly must be able to be stopped precisely at any graduation mark on the column, and

4. The movement from one elevation to another must be as rapid as possible, keeping in mind the accuracy of positioning required by (3).

A general view of the soil column in relation to the entire experimental arrangement may be seen in Figures 3.1 and 3.2.

3.3.2 Water Content Measurements

The full and desirable specification for water content measurements for the unsaturated transient flow types of problem is:

1. measurement over a small thickness of the sample to approximate as closely as possible a planar measurement,

2. determination by nondestructive means,

3. measurement over a very short time interval, and

4. rapid means of making measurements at different parts of the soil column.

This specification is extremely exacting and has not been met in any method until recently. However, such workers as Ferguson (107), Rawlings
(108) and Gurr (109) have shown that the principle of gamma-ray absorption may be used to infer the moisture content of the soil from changes in the soil density for those soils whose pore geometry remains unchanged during water content variations. This principle enables some of the most difficult aspects of the specification to be satisfied with allied instrumentation completing the requirements.

The attenuation equation for a moist soil and for a collimated, monoenergetic beam is as follows:

\[
\frac{I_m}{I_0} = \exp \left( -\left( \mu_b \cdot \rho_b \cdot \theta + \mu_s \cdot \rho_s \right) \right) \times x \tag{3-5}
\]

where

- \( \frac{I_m}{I_0} \) = ratio of incident to transmitted flux for moist soil,
- \( \mu_b \) & \( \mu_s \) = mass attenuation coefficient for water and soil (cm\(^2\)/g),
- \( x \) = thickness of soil (cm),
- \( \theta \) = volume of water/unit volume (cc/cc),
- \( \rho_s \) = bulk density of the soil (g/cc), and
- \( \rho_b \) = density of water (g/cc).

A study of Equation (3-5) reveals that its use in determining \( \theta \) requires:

1. values of the mass attenuation coefficients - these are conveniently determined using a specially designed sectioned box as described by Watson (110),
2. a uniform and known bulk density of the soil in the column - this can be achieved by special packing techniques,
3. a constant thickness dimension along the column length - this is dependent upon the column construction and requires fabrication with the fine tolerances.
A Cesium 137 source, a scintillation probe, and a scaler unit are used in a nondestructive gamma radiation technique to determine changes in moisture content in the soil along the length of the column, Figure 3.3. The 137Cs source and scintillation probe-preamplifier unit, along with the required lead shielding, are moved along the length of the soil column by use of three threaded screws which are part of the supporting frame. The scintillation probe, which absorbs the gamma radiation after it has passed through the soil column is connected to a decade scaler and pulse-height analyzer. Moisture content or density are determined from the number of counts registered by the decade scaler.

3.3.3 Soil Water Pressure Measurement

There are two aspects of pressure measurement which must be embraced in any experimental program relating to unsteady unsaturated porous materials; the first concerns the pressure changes in soil water during water movement and the second relates to the air pressure increase which occurs in bounded or effectively bounded columns during wet front advance.

A tensiometer system for the measurement of rapidly changing suction should have the following features:

1. a high degree sensitivity,
2. a response time of the order of few seconds or less,
3. rapid speed of reading,
4. the convenience of recording.

The achievement of a very rapid response in the tensiometer system to suction changes in the soil inherently implies a negligibly small exchange of water between the soil and the tensiometer. Klute and Peters (111) and Watson (112) previously reported the satisfactory use of a pressure
transducer for soil water measurements requiring rapid response with minimum transfer of water between the soil-water and the measuring system. This approach has been developed in the present tensiometer-transducer system and is undoubtedly ideal for use in suction range up to 900 cm of water.

A porous cup with as large a pore size as possible, yet with an air entry value greater than the maximum suction to be encountered, should be used to increase the response of the measuring system. The ceramic porous cups (fine porosity) used for this study reported here as tensiometers were 1.0 cm in diameter and 3.0 cm in length with an air entry value of approximately 800 cm of water. Tensiometers were installed at 8 positions and were saturated with boiled, distilled water. All tensiometers were connected with 1/16 in. O. D. nylon tubing to a single pressure transducer (Validyne Model DP15T1) by way of a common rotary switching valve (Scanivalve Model W0601/IP-24T). The output signal of the pressure transducer was measured by a Validyne Carrier Demodulator (Model CD 23) and continuously recorded on a strip chart recorder.

A constant head water column was used as a standard to calibrate the transducer at 0, -10, -20, -40, -60, -100, -120 cm pressure head. The ratio of signal voltage to differential pressure used for this study was 1mV/cm of cm of water pressure head.

The response time-constant of the tensiometer-transducer system when the flow system was water saturated was 0.5 seconds but the response became quite large (< 20 seconds) when the flow system was in an unsaturated condition. Rapid decrease of hydraulic conductivity of the soil with decreasing water content caused an increase in the time required for the transmission of water through the porous medium to or from the tensiometer.
Richards (113) defined the tensiometer conductance, \( k \), as the volume of water passing through the tensiometer cup per unit of time per unit of pressure difference. For a given time and pressure difference, the cup conductance depends primarily on the area of contact with soil and the pore size of its porous material. Also, he defined the gauge sensitivity, \( s \), as the pressure change per unit volume of displacement. Watson (112) stated that the gauge sensitivity, for the equipment under discussion, could more precisely be described as transducer sensitivity. The response time-constant is related to the tensiometer cup conductance, \( k \), and the gauge sensitivity, \( s \), according to Richards (113) by the equation:

\[
\tau = \frac{1}{ks}
\]  

(3-6)

where \( \tau \) is the response-time constant (sec.); \( k \), the tensiometer conductance (\( \text{cm}^3/\text{sec/atm} \)); and \( s \), the gauge sensitivity (atm/cm\(^3\)).

Since the manufacturer specified that the pressure transducer used throughout this study has a volumetric displacement of \( 4.92 \times 10^{-3} \text{cm}^3 \), then, for a maximum pressure of 850 cm of water and gauge sensitivity value of \( 1.7 \times 10^5 \text{cm}^{-3} \), it can be seen that the tensiometer cup conductance might be easily calculated from Equation (3-6) for a specified response time-constant.

3.4 Laboratory Study Results

A value of \( 0.0832 \pm 0.0006 \text{ cm}^2/\text{g} \) for the gamma-ray mass adsorption coefficient for water, \( \mu_\theta \), was used in water content calculations. This value was in agreement with the theoretical (Grodstein(114)) value of \( 0.0857 \text{ cm}^2/\text{g} \). The gamma-ray mass adsorption coefficient for soil, \( \mu_s \), of
AASHO A-4 (Illinois Till C) was measured to have a value of $0.0760 \pm 0.0001$ cm$^2$/g. The calculations were done, using an average bulk density, $\gamma_d$, value of 1.72 g/cm$^3$ ($\gamma_d = 107.3$ pcf) and optimum soil water content, $W_{opp}$ of 11.7%, as reported by Gurr (109).

Selected physical properties of the AASHO A-4 soil are presented in Table 3.1. Particle size distribution showed a predominance of sand (more than 60%) fraction, however, the soil clay content (18%) and its compacted nature ($\gamma_d = 1.72$ g/cm$^3$) would explain the low value of the saturated hydraulic conductivity, $K_s$.

Using the instantaneous profile method as suggested by Watson (115) and used by Elzeftawy and Mansell (116), the measured unsaturated hydraulic conductivity as a function of soil water content, $K(\theta)$, and soil water content-suction characteristic curve, $h(\theta)$, for transient flow were determined. Figure 3.4 shows the water content on a volume basis as a function of soil water suction expressed as cm of water head for AASHO A-4 soil. The solid line was eye-fitted to connect all the measured values of $h(\theta)$ relationship. It should be mentioned that the data presented in Figure 3.4 is for the case of transient water flow during the wetting of the soil column. Notice that a value of 1.0 cm of water suction head was considered to be the value at which the soil was saturated. The AASHO A-4 soil water content at saturation, $\theta_s$, was 0.44 in comparison to 0.28 cm$^3$/cm$^3$ at 2066 cm of soil water suction head, $h$. In other words, the soil has lost approximately 36% of its water content in response of applying 2066 cm of water pressure head.
Campbell (117) indicated that if the water characteristic function, \( h(\theta) \), can be represented by the equation

\[
h = h_e (\theta/\theta_s)^{-b}
\]  

(3-7)

where \( h_e \) is the air entry water potential and \( b \) is an empirically determined constant, then the hydraulic conductivity is given by

\[
K = K_s (\theta/\theta_s)^{2b+3}
\]  

(3-8)

where \( K_s \) is the saturated hydraulic conductivity of the soil. Since Equation (3-7) is assumed to describe the water characteristic curve for the AASHO A-4 soil, the data points of Figure 3.4 plotted on a log-log scale should produce a straight line with slope equal to \(-b\). A value of \( b=13.1 \) was obtained from a least squares fit of a straight line to the data of Figure 3.4 over the range indicated by data points in Figure 3.5. If this condition is not met, Equation (3-8) cannot be expected to give valid estimates of \( K(\theta) \).

The measured and calculated unsaturated hydraulic conductivity of AASHO A-4 soil are shown in Figure 3.6. The calculated \( K(\theta) \) was obtained by using Equation (3-8) of Campbell (117) and Elzeftawy and Mansell (116) method. The Elzeftawy and Mansell method was obtained by modifying Green and Corey (118) method to include a spline function (Erh (119)) technique to provide smooth continuous soil water characteristic function. This method was successfully used by Elzeftawy and Mansell (116) to determine the unsaturated hydraulic conductivity of Lakeland fine sand soil. The measured value of hydraulic conductivity corresponding to water saturation (\( \theta_s = 0.44 \text{ cm}^3/\text{cm}^3 \)) was used as a matching factor to determine the calcu-
lated curve for $K(\theta)$ function. Matching at water saturation has a distinct advantage over matching at desaturated water contents since inaccuracies in calculated curves can be more easily tolerated at lower water contents than that at high water contents when calculated $K(\theta)$ results are to be used in subsequent prediction of infiltration or evaporation (Green and Corey, [118]). Determinations of water-saturated conductivities are also much simpler and quicker to determine experimentally than unsaturated conductivities. Agreement of $K$ calculated using Equation (3-8) with experimentally determined conductivities of AASHO A-4 soil was found to be good, however; departures from the equation can be expected in the wet range (> -0.1 bar) because the water potential approaches the air entry value and the water content approaches saturation. It can be seen from Figure 3.6 that the revised method of calculating $K$ versus $\theta$ (method of Elzeftawy and Mansell), based on Green and Corey's method, offers a better agreement with the measured values of the unsaturated hydraulic conductivity than that of Campbell [117]. The divergence of the Campbell method from the measured conductivity may be referred to the assumption that the pore size distribution function is the same throughout the porous body. In any case, agreement between calculated and measured hydraulic conductivities appears to be good enough for most practical purposes.

Just as the flow of heat expressed in the form of a diffusion equation with a diffusivity expressed in terms of thermal conductivity, density, and specific heat of material, so in certain circumstances may Darcy's law be put into a diffusion-like form with hydraulic diffusivity $D(\theta)$ given by

$$D(\theta) = \frac{K(\theta)}{C(\theta)}$$  (3-9)
The denominator, \( C(\theta) = \frac{3h}{3h} \), is the specific water capacity of the soil. Figure 3.7 shows the AASHO A-4 water diffusivity as a function of soil water content, \( \theta \). Since soil water diffusivity, \( D(\theta) \) is a hysteretic function, i.e., \( D(\theta) \) depends upon the wetting or drying the soil water system, it should be mentioned that the data presented in Figure 3.7 is for the wetting of AASHO A-4 soil and it should be used only for wetting cycles of water movement. Notice that by increasing the soil water content, \( \theta \), from 0.28 to 0.42 cm³/cm³, the soil water diffusivity has increased from \( 3.2 \times 10^{-1} \) to \( 5.4 \times 10^{1} \) cm²/h, respectively. For the range of increasing the water content, soil hydraulic conductivity increased from \( 3.1 \times 10^{-7} \) to \( 1.9 \times 10^{-1} \) cm/h, respectively (Figure 3.6).

Recent studies conducted by Robnett and Thompson (120) and Robnett (121) with a large number of soils from Midwest, Oklahoma, Georgia, and the Carolinas, have demonstrated the dramatic influence that moisture content has on the resilient behavior of soils. They indicated that 1 or 2 percent water content change can have considerable influence on the strength of the AASHO Road Test subgrade soil.

The experimentally determined relationships of \( h \) versus \( \theta \) (Figure 3.4), \( K \) versus \( \theta \) (Figure 3.6), and \( D \) versus \( \theta \) (Figure 3.7) were used in the numerical mathematical solutions of Equation (3-4) under isothermal conditions to predict the moisture movement from the established water table (W.T.) at the bottom of the soil column. The moisture model which can be utilized for one-dimension or two-dimension moisture flow through homogeneous or multilayered subgrade soil and pavement systems is described in section 4 of this report.
4.1 Introduction

In recognition of the need for developing a reasonable and realistic procedure for evaluating moisture conditions in pavement systems, research was undertaken with the following objectives:

1. Develop, based on climatic factors, a theoretical model for predicting moisture movement and moisture equilibria in pavement systems.

2. Validate the moisture model by means of available laboratory data.

4.2 Development of Mathematical Moisture Model

A review of the literature indicated that the moisture movement theory presented by Philip and de Vries (64) provided the most comprehensive basis for the development of a moisture model to predict transient flow in pavement systems. By differentiating Equation (2-26) and applying the continuity requirement, the general partial differential equation describing moisture movement in porous materials under combined temperature and moisture gradients can be stated as follows:

\[ \frac{\partial \theta}{\partial t} = \nabla \cdot (D_T \nabla T) + \nabla \cdot (D_\theta \nabla \theta) + \frac{\partial K_\theta}{\partial z} \]  

(4-1)

From Philip and de Vries (64) the equation describing heat transfer is as follows:

\[ \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T - L \nabla \theta - D_\text{evap} \nabla \theta) \]  

(4-2)
Equations (4-1) and (4-2) are the basic equations used to describe moisture movement and heat-transfer in the mathematical moisture model. (symbols used in all equations are defined in Appendix A).

In Equation (4-1), the thermal diffusivity, \( D_T \), has two components which can be expressed by the following equation:

\[
D_T = D_{T\text{liq}} + D_{T\text{vap}}
\]

(4-3)

Similarly the moisture diffusivity, \( D_\theta \), has two components as follows:

\[
D_\theta = D_{\theta\text{liq}} + D_{\theta\text{vap}}
\]

(4-4)

The two liquid diffusivities tend to be the most important at high moisture contents while the two vapor diffusivities are dominant at low moisture contents (64).

4.2.1 Finite Difference Approximation for Water Movement and Heat-Transfer Equations

In the moisture model the water flow equation [Equation (4-1)] and the heat-transfer equation [Equation (4-2)] are nonlinear second-order parabolic partial differential equations. Since exact solutions to the nonlinear equations are very difficult and sometimes impossible to obtain, numerical methods were used to develop the moisture model.

In the moisture model the numerical solutions to Equations (4-1) and (4-2) are obtained by first expressing them as finite difference approximations. The three main types of methods commonly used to provide finite difference approximations are fully implicit, fully explicit, and implicit-explicit methods. The moisture model was developed by using the implicit finite difference approximation, Figure 4.1. The implicit method is always stable.

The implicit finite difference approximation of the water flow equation [Equation (4-1)] can be expressed as follows for the one-dimensional case:
\[
\frac{\theta_{i}^{n+1} - \theta_{i}^{n}}{\Delta t} = D \frac{(T_{i+1/2}^{n+1} - T_{i+1/2}^{n})}{(\Delta z)^{2}} \cdot (T_{i+1}^{n} - T_{i}^{n}) \\
- D(T_{i-1/2}^{n+1} - T_{i-1/2}^{n}) \frac{T_{i}^{n} - T_{i-1}^{n}}{(\Delta z)^{2}} \\
+ D(\theta_{i+1/2}^{n+1/2} - \theta_{i+1/2}^{n+1}) \frac{\theta_{i+1}^{n+1} - \theta_{i}^{n+1}}{(\Delta z)^{2}} \\
- D(\theta_{i-1/2}^{n+1} - \theta_{i-1/2}^{n+1}) \frac{\theta_{i}^{n+1} - \theta_{i-1}^{n+1}}{(\Delta z)^{2}} \\
- K(\theta_{i+1/2}^{n+1/2} - \theta_{i-1/2}^{n+1/2}) \frac{\theta_{i+1}^{n+1/2} - \theta_{i-1}^{n+1/2}}{\Delta z} \tag{4-5}
\]

Similarly the finite difference approximation for the heat-flow equation [Equation(4-2)] can be expressed as follows for the one-dimensional condition:

\[
C \frac{T_{i}^{n+1} - T_{i}^{n}}{\Delta t} = \frac{\lambda}{\Delta t} \cdot (T_{i+1/2}^{n+1} - T_{i+1/2}^{n}) \frac{T_{i+1}^{n+1} - T_{i}^{n+1}}{(\Delta z)^{2}} \\
- \frac{\lambda}{\Delta t} \cdot (T_{i-1/2}^{n+1} - T_{i-1/2}^{n}) \frac{T_{i-1}^{n+1} - T_{i}^{n+1}}{(\Delta z)^{2}} \\
- L D_{\text{evap}}(\theta_{i+1/2}^{n+1/2} - \theta_{i+1/2}^{n}) \frac{\theta_{i+1}^{n} - \theta_{i}^{n}}{(\Delta z)^{2}} \\
- L D_{\text{evap}}(\theta_{i-1/2}^{n+1/2} - \theta_{i-1/2}^{n}) \frac{\theta_{i}^{n} - \theta_{i-1}^{n}}{(\Delta z)^{2}} \tag{4-6}
\]

In the moisture model the numerical solution to Equations (4-5) and (4-6) gives the water content \( \theta(z,t) \) and temperature \( T(z,t) \) at incremental distances, \( \Delta z \), and incremental times steps, \( \Delta t \), where \( z = i \Delta z \) and \( t = n \Delta t \).
Equation (4-5) can be solved for the water content, $\theta_i^{n+1}$, at time step equal to $n+1$, where $\theta_i^n$ and $T_i^n$ are known from the initial boundary conditions. Equation (4-6) can be solved for the temperature, $T_i^{n+1}$, at time step $n+1$ by using the water content computed from Equation (4-5) and values of $T_i^n$. Repeated solutions of Equations (4-5) and (4-6) provide the water content and temperature distribution at any particular time desired. In the moisture model the temperature computation lags behind the moisture computation by one time step.

The numerical solution to the implicit finite difference equations results in a set of simultaneous equations which can be solved by the Gauss elimination method.

In the moisture model and temperature and moisture diffusivities each has two components as described by Equations (4-3) and (4-4). The procedure for obtaining the liquid moisture diffusivity, $D_{\theta \text{liq}}$, has been described in Section 3 of this report where $D_{\theta \text{liq}} = K \frac{\alpha h}{\alpha \theta}$. The vapor moisture diffusivity, $D_{\theta \text{vap}}$, was obtained from the following equation presented by Philip and de Vries (64):

$$D_{\theta \text{vap}} = D_{\text{atm}} \vee \alpha a g \rho \left( \frac{\alpha h}{\alpha \theta} \right) / \rho \text{w} \cdot \text{RT} \quad (4-7)$$

Also from Philip and de Vries (64) the thermal liquid diffusivity, $D_{\text{Tliq}}$, and thermal vapor diffusivity, $D_{\text{Tvap}}$, were computed from the following equations:

$$D_{\text{Tliq}} = K \gamma h \quad (4-8)$$

and

$$D_{\text{Tvap}} = D_{\text{atm}} \vee \alpha a h_0 \beta / \rho \quad (4-9)$$
4.2.2 Computer Program

The numerical solution to the implicit finite difference equations (Equations 4-5 and 4-6) was programmed on a digital computer to provide a comprehensive and flexible moisture model. Appendix B lists the typical inputs to the model. From Appendix B it can be noted that one-dimensional or two-dimensional studies can be conducted. It is also possible to specify whether isothermal or nonisothermal moisture movement is to be predicted in the pavement system. Stationary or moving water table conditions can also be specified in the moisture model. The data are programmed into the computerized moisture model by using a free-form scan program. This procedure allows substantial flexibility in changing various inputs to the computer program.

Climatic conditions can be put into the moisture model by using standard weather bureau data. Table 4.1 shows typical weather data which is used in the moisture model.

Most of the temperature computation procedures used in the moisture model were based on a heat-transfer model developed by Dempsey (122). Meteorological parameters such as radiation and convection into or out of the pavement system are considered, Figure 4.2. A meteorological energy balance approach previously used by Dempsey (122) is used to relate the climatic parameters to the pavement surface as follows:

\[ Q_i - Q_f - Q_a - Q_e + Q_c + Q_h + Q_g = 0 \]

Precipitation at the pavement surface is considered in the moisture model either as snow or rain. The amount of rain water which infiltrates the pavement is a function of the rainfall intensity and duration, surface runoff, and pavement surface permeability. Water infiltration from snow is considered only if the temperature rises above freezing.
4.3 Validation of Moisture Model

Validation of the moisture model using data from controlled laboratory and field studies is being conducted as part of this study.

The field investigations are not completed at this time, however, comparisons between predicted and measured moisture contents have been made using laboratory data from one-dimensional studies.

Figure 4.3 shows the comparisons between predicted and measured moisture contents with depth in Lakeland fine sand (AASHO Classification A-3) for one-dimensional, isothermal conditions.

The physical properties of the Lakeland fine sand are listed in Table 4.2. The boundary conditions for the program were as follows:

1. The moisture content at the surface remained constant at 0.26 cm$^3$/cm$^3$ (gravimetric water content = 16.6%) during $0 \leq t \leq t_{\text{final}}$.
2. The water table was sufficiently deep so that the moving water front did not reach to that depth.
3. The average initial water content distribution at $t = 0$ was 0.109 cm$^3$/cm$^3$ (gravimetric water content = 7.0%).

From Figure 4.3 it is apparent that the isothermal moisture contents predicted by the moisture model compare favorably with the measured moisture contents with time.

Figure 4.4 shows a comparison between predicted and measured isothermal moisture contents for an Illinoian till soil (AASHO Classification A-4) for a water table condition. The physical properties of the Illinoian till are presented in Table 3.1. The boundary conditions for the program were as follows:
1. The average initial water content distribution at \( t = 0 \)
was 0.21 \( \text{cm}^3/\text{cm}^3 \) (gravimetric water content = 12.2%).

2. An instantaneous water table moved upward to a depth
of 120 cm (3.94 ft) below the surface at \( t = 0 \).

3. The surface moisture was not allowed to evaporate
during \( 0 \leq t \leq t_{\text{final}} \).

In Figure 4.4 it would appear that the moisture model was adequate in
predicting moisture content changes with time in a fine grained soil for
one-dimensional, isothermal conditions. By comparing Figure 4.3 with Figure
4.4 it is possible to observe the differences in the rate of moisture move-
ment caused by soil type and force of gravity. In Figure 4.3 the downward
movement of water is assisted by gravity while in Figure 4.4 the upward
migration of water from the water table must move against the gravity force.
From Figure 4.4 it would appear that more than 60 days may be required for
the Illinoian till soil to reach equilibrium for the given boundary condi-
tions.

Nonisothermal moisture data are not readily available at this time to
validate the moisture model. However, Figure 4.5 shows the influence of a
temperature gradient of approximately 0.2 \( \text{C/cm} \) (0.9 \( \text{F/in.} \)) on the Illinoian
till soil for the same initial boundary conditions as specified for Figure
4.4. The temperature increased from the top downward. It can be noted from
comparing Figures 4.4 and 4.5 that temperature can exert substantial influ-
ence on moisture movement in a soil with time. It would appear from the
relative influence of temperature on moisture movement that the model has
the potential to simulate nonisothermal conditions.

4.4 Discussion of Moisture Model

The moisture model represents a comprehensive and flexible method for
predicting moisture conditions in pavement systems with time. As additional laboratory and field data become available, further validation work will be accomplished for both one-dimensional and two-dimensional moisture movement.

The ultimate objective of the application of moisture flow theory to field flow situations is to understand the soil water regime for the benefit of pavement design. In the process of developing the theoretical concepts that model the soil-water flow system, there is an increased understanding of the system. Theoretical concepts can in principle be used in making predictions of the response of a soil-water flow system to the imposition of particular boundary conditions or modifications of the properties of the system. By making an adequate prediction it is them possible to modify or manage the behavior of the soil-water system from the unsaturated to saturated range.

In a broad way there would seem to be two purposes for the quantitative analysis and prediction of the performance of a given flow system. They are the verification of the validity of the flow theory, and the practical prediction of the hydraulic performance of a given body of soil material in the pavement system.

The theory of soil water movement is often used in a qualitative manner. As the basic concepts of flow in unsaturated soils become more generally known and understood by engineers, it is evident that more use can and will be made of these concepts. As an example, the recognition that the hydraulic conductivity decreases rapidly with decreasing water content can be of great use in qualitative and quantitative ways to analyze the behavior of a soil-water system.
It is generally felt that the moisture model represents an important and necessary step in the development of a better understanding of climatic effects on pavement systems. Stating the moisture flow theory in mathematical terms as embodied in the partial differential equations of flow provides a basis for a more quantitative prediction of behavior. The classical mathematical-physical approach requires mathematical statements of the initial and boundary conditions that describe the specific flow situations and knowledge of the conductivity and water capacity functions that characterize the soil. It is then possible to obtain a solution of the flow equation and predict the behavior of the flow system using analytical or numerical methods. The solution is in the form of the spatial and temporal distribution of the water content and/or the pressure head of the soil water. From these, such quantities as flux and cumulative flows can be derived at any point in the pavement system.

4.5 **Applications of Moisture Model**

Research has indicated that moisture is an important factor effecting the durability properties and resilient properties of highway soils and materials, and the performance and fundamental behavior of pavement systems. Water directly governs the mechanical properties of most pavement materials and soils; therefore, any variation in water content will alter the properties of most pavement materials and soils. From materials research it has become apparent that methods for predicting moisture movement and moisture equilibria in pavement systems are needed in order to fully describe material
and pavement behavior. The increased use of subsurface drains also supports the need for a better understanding of moisture movement and moisture equilibria in pavement systems. It is evident that sound decisions concerning the location of subsurface drains require a thorough knowledge of time-dependent moisture regime of the pavement system.

The moisture model can be used to study the changes in moisture content in a pavement system with time for varying climatic conditions. Figures 4.6 and 4.7 show variations in subgrade moisture content and temperature respectively for a 6 day period with fluctuating air temperatures and with precipitation, Figure 4.8. The physical properties of the soil used to develop Figures 4.6 and 4.7 are the same as those for the Illinoian till in Table 3.1. The climatic input data was selected from a weather station in northern Illinois. It can be noted that the subgrade moisture changes shown in Figure 4.6 reflect the influence of the rainfall shown in Figure 4.8. Although not readily evident, the water table position and temperature also have an effect on the moisture changes noted with time in Figure 4.6.

It is expected that the findings from this investigation will be helpful in making decisions related to moisture problems in pavement systems. The moisture model can be used to determine how various design modifications influence the moisture regime in a pavement system. It is anticipated that the model and related field and laboratory studies will lead to a less empirical approach for incorporating moisture effects into pavement design, construction, behavior, and performance.

4.6 Summary and Conclusions

4.6.1 Summary

An investigation was conducted to develop a comprehensive moisture model for predicting moisture conditions in pavement systems. An extensive
literature review indicated that a model based on the Philip and de Vries equations for nonisothermal moisture movement and heat conduction would give the best results. By using numerical methods, the implicit finite difference approximations to the moisture movement and heat-transfer equations were programmed for computer solution of water content and temperature in pavement systems with time.

Validation studies indicated that the moisture model could be used to accurately predict isothermal moisture conditions in pavement systems. The moisture model was also used to show the relative influence of nonisothermal conditions on pavement moisture content. The moisture model was shown to be applicable to a wide range of boundary conditions and that it could be used to predict the moisture-temperature regime with time in pavement systems utilizing climatic input data.

Further studies to validate the moisture model for one-dimensional and two-dimensional field and laboratory conditions are in progress. These studies will include both isothermal and nonisothermal moisture measurements and predictions.

4.6.2 Conclusions

From the results of this investigation, the following conclusions were made:

1. The moisture model provides a comprehensive procedure for predicting moisture conditions in pavement systems.
2. The Philip and de Vries equation for moisture movement provides a sound basis for predicting moisture conditions in pavement systems.
3. The moisture model gives valid results for predicting isothermal moisture movement.
4. Although not validated at this time, the moisture model displays the potential for predicting nonisothermal water movement.

5. The moisture-temperature regime in pavement systems can be predicted with time based on climatic input data.

6. The moisture model displays potential use for many types of pavement research related to climatic effects.
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Table 2.1. Potential Expressed in the Major Measurement Systems.

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Table 2.2. Conversions for Various Potential Units

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<td>33.9</td>
<td>1013000</td>
<td>101.3</td>
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Table 3.1. Selected Physical Properties of Illinoian Till  
(AASHO Classification A-4)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
<tr>
<td>Sand (%)</td>
<td>62.00</td>
</tr>
<tr>
<td>Silt (%)</td>
<td>20.00</td>
</tr>
<tr>
<td>Clay (%)</td>
<td>18.00</td>
</tr>
<tr>
<td>Liquid Limit, LL (%)</td>
<td>22.20</td>
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<tr>
<td>Plastic Limit, PL (%)</td>
<td>14.70</td>
</tr>
<tr>
<td>Compacted Dry Density, $\gamma_d$ (g/cm$^3$)</td>
<td>1.72 (107.3 pcf)</td>
</tr>
<tr>
<td>Compacted Water Content, $w$ (%)*</td>
<td>11.70</td>
</tr>
<tr>
<td>$w_{HYGR}$ (%)*</td>
<td>1.40</td>
</tr>
<tr>
<td>Saturated Hydraulic Conductivity, $K_s$ (cm/sec)</td>
<td>$8.61 \times 10^{-5}$ (0.01 ft/hr)</td>
</tr>
</tbody>
</table>

*Gravimetric Water Content
Table 4.1. Weather Input Data

<table>
<thead>
<tr>
<th>Year of study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month of study</td>
</tr>
<tr>
<td>Day of month</td>
</tr>
<tr>
<td>Maximum daily temperature</td>
</tr>
<tr>
<td>Minimum daily temperature</td>
</tr>
<tr>
<td>Rainfall</td>
</tr>
<tr>
<td>Snowfall</td>
</tr>
<tr>
<td>Wind velocity</td>
</tr>
<tr>
<td>Percentage of possible daily sunshine</td>
</tr>
<tr>
<td>Time of sunrise</td>
</tr>
<tr>
<td>Time of sunset</td>
</tr>
<tr>
<td>Theoretical extraterrestrial radiation</td>
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Table 4.2. Selected Physical Properties of Lakeland Fine Sand  
(AASHO Classification A-3)

<table>
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<th>Property</th>
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<tr>
<td>Sand (%)</td>
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<tr>
<td>Silt (%)</td>
<td>2.00</td>
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<tr>
<td>Clay (%)</td>
<td>0.00</td>
</tr>
<tr>
<td>Liquid Limit, LL (%)</td>
<td>N.P.</td>
</tr>
<tr>
<td>Plastic Limit, PL (%)</td>
<td>N.P.</td>
</tr>
<tr>
<td>Compacted Dry Density, $\gamma_d$ (g/cm$^3$)</td>
<td>1.56 (97.3 pcf)</td>
</tr>
<tr>
<td>Compacted Water Content, w (%)*</td>
<td>0.65</td>
</tr>
<tr>
<td>HYGR (%)*</td>
<td>0.65</td>
</tr>
<tr>
<td>Saturated Hydraulic Conductivity, $K_s$ (cm/sec)</td>
<td>$4.11 \times 10^{-3}$ (0.48 ft/hr)</td>
</tr>
</tbody>
</table>
Figure 1.1. Research Needs for Prediction and Control of Moisture in Pavement Systems (Reference 1).
Figure 1.2. Approach for Predicting Moisture in Pavement Systems (Reference 1).
Figure 1.3. Methods of Controlling Moisture in Pavement Systems (Reference 1).
Figure 2.1. Sources of Moisture in Pavement Systems (Reference 3).
Figure 2.2. Variations in Subgrade Moisture Contents with Time (Reference 6).
Figure 2.3. Variations in Subgrade Moisture Contents Compared with Fluctuations in Ground Water Table (Reference 6).
Figure 2.4. Moisture Content Variations for Airfield Pavements (Reference 8).
Figure 2.5. Effect of Subgrade Saturation on Pavement Deflection - WASHO Road Test (Reference 18).
Figure 2.6. AASHO Road Test - Spring and Summer Subsurface Conditions - Loop 1, No Traffic (Reference 19).
Figure 2.7. AASHO Road Test - Subbase Condition Data, Spring and Summer, 1960 (Reference 19).

Figure 2.8. AASHO Road Test - Base Course Conditions, Spring and Summer, 1960 (Reference 19).
Figure 2.9. AASHO Road Test - Embankment Condition Data, Spring and Summer, 1960 (Reference 19).
Figure 2.10. Effect of Compaction Moisture on Resilient Modulus (Reference 20).
Pavement Conditions

Portland Cement Concrete; Improved Shoulder, Sand Subbase. Excellent Condition

Figure 2.11. Subgrade Moisture Content Variations at the Pavement Centerline (Reference 9).
Figure 2.12. Typical Seasonal Variations in Deflection and Curvature (Reference 27).
Figure 2.13. Seasonal Changes in Bearing Capacity (Reference 33).
Figure 2.14. Seasonal Relationship Between Moisture and Deflection (Reference 35).
Figure 2.15. Moisture and Density Effects for the AASHO Road Test Embankment Soil (Reference 38).

NOTES
1. Figure beside curve is molding water content
2. Surcharge equal 20 lb soaking and penetration
3. All samples soaked 4 days
4. All samples compacted in 5 layers, 10 lb hammer, 18-inch drop in CBR mold

LEGEND

△ 55 Blows per layer
● 26 Blows per layer
○ 12 Blows per layer
Figure 2.16. Influence of Soil Suction on Shear Strength (Reference 39).
Figure 2.17. Effect of Suction on the CBR of a Silty Sand (Reference 39).
Figure 2.18. Effect of Soil Suction on the Resilient Modulus of a Canadian Till (Reference 40).

- Deviator Stress = 10 psi
- Confining Pressure = 3.5 psi
- Frequency Of Stress
  - Application = 20 per min
- Duration Of Stress
  - Application = 0.25 sec
- Dry Density = 109.6 lb per cu ft
Figure 2.19. Seasonal Relationship Between Moisture and Deflection for a Subgrade Soil (Reference 35).
Figure 2.20. Effect of Degree of Saturation on the Repeated Load Deformation Properties of the AASHO Granular Materials (Reference 42).
Figure 2.21. Capillary Tubes Showing the Configuration of the Air-Water Interfaces at Different Heights.
Figure 2.22. Variation of Pressure Above and Below the Water Table.
Figure 2.23. Soil-Water Characteristic Curves (Reference 53).
Figure 2.24. Simple Tensiometer System.
Figure 2.25. Soil-Water Characteristic Curve Showing Hysteresis (Reference 51).
Figure 2.26. Hydraulic Conductivity vs Matric Potential for Several Major Soil Types (Reference 61).
\[ W_n = \text{water content} \]
\[ LP = \text{plastic limit} \]

1. \[ W_n = LP \quad (L.C.P.C.) \]
2. \[ W_n = 1.16 \cdot LP - 7.6 \quad (\text{Swanberg}) \]
3. \[ 0.87 \cdot LP < W_n < 1.11 \cdot LP \quad (L.T.M.S.) \]
4. \[ W_n < LP + 2 \quad (U.S. \text{ Navy}) \]
5. \[ 0.8 \cdot LP < W_n < 1.2 \cdot LP \quad (\text{Kersten}) \]
6. \[ W_n = 1.17 \cdot LP - 4 \quad (\text{Wooltorton}) \]

Figure 2.27. Empirical Relationships Between Plastic Limit and Moisture Content (Reference 1).
Figure 2.28. British Road Research Laboratory Method for Determining Equilibrium Moisture Content in Subgrade Soil (Reference 1).
Figure 2.29. Transient Water Content Distribution, $\theta$, for Ida Silt Loam with Impervious Barrier after 120 Minutes (Reference 94).
Figure 2.30. Transient Water Content Distribution, $\theta$, for Webster Clay Loam with Ground-Water Table after 120 Minutes (Reference 94).
Figure 2.31. Steady State Water Content Distribution, $\theta$, for Ida Silt Loam with Impervious Barrier (Reference 94).
Figure 2.32. Steady State Water Content Distribution, $\theta$, for Webster Clay Loam with Groundwater Table (Reference 94).
Figure 2.33. Laboratory Comparison between Calculated and Experimental Soil-Water Profiles in Lakeland Fine Sand (Reference 97).
Figure 2.34. Field Comparison between Calculated and Experimental Soil-Water Profiles in Lakeland Sand (Reference 97).
Figure 2.35. Comparison Between Measured and Computed Moisture Contents at a Field Test Site in Wyoming (Reference 95).
Figure 3.1. Laboratory Soil Column and Equipment

Figure 3.2. Laboratory Instrumentation
Figure 3.4. Soil Water Characteristic Function for AASHO A-4 Subgrade Soil.
Figure 3.5. Water Retention (Wetting) Curve for AASHO A-4 Subgrade Soil Used to Determine Value for b. See Text for Sources of Data.
Figure 3.6. Unsaturated Hydraulic Conductivity of AASHO A-4 Subgrade Soil Calculated by Several Procedures Over a Range of Water Contents in Comparison to Measured Values.
Figure 3.7. Soil Water Diffusivity for AASHO A-4 Subgrade Soil as a Function of Water Content for a Wetting Case.

\[
D(\theta) = \frac{K(\theta)}{C(\theta)}
\]

\[
C(\theta) = \frac{\delta \theta}{\delta h}
\]
Figure 4.1. Implicit System for Diffusion Equation.
Figure 4.2. Heat Transfer Between Pavement and Air (Reference 122).
Figure 4.3 Comparison between Predicted and Measured Moisture Contents for Lakeland Fine Sand with a Constant Water Source at the Surface.
Figure 4.4 Comparison between Predicted and Measured Moisture Contents for Illinoian Till Above a Water Table.
Figure 4.5. Predicted Nonisothermal Moisture Movement for Illinoian Till Above a Water Table.
Figure 4.6. Changes in Nonisothermal Pavement Moisture Content as a Function of Depth, Climate, and Time.
Figure 4.7. Changes in Pavement Temperature as a Function of Depth, Climate, and Time.
Figure 4.8. Air Temperature and Precipitation Input for Moisture Model.
APPENDIX A

Identification of Symbols

a  Volumetric air content, cm³/cm³
C  Volumetric heat capacity, Cal/cm³/C
D_θ  Isothermal moisture diffusivity, cm²/sec
D_T  Thermal moisture diffusivity, cm²/sec/C
D_eliq  Isothermal liquid diffusivity, cm²/sec
D_evap  Isothermal vapor diffusivity, cm²/sec
D_Tliq  Thermal liquid diffusivity, cm²/sec/C
D_Tvap  Thermal vapor diffusivity, cm²/sec/C
D_atm  Molecular diffusion coefficient of water vapor in air, cm²/sec
G  Acceleration due to gravity, cm/sec²
H  Total water potential, cm
h  Capillary potential, cm
h₀  Relative humidity
K and K_θ  Unsaturated hydraulic conductivity, cm/sec
L  Heat of vaporization, Cal/g
q and Q  Water flux, cm/sec
Q_a  Heat flux resulting from long-wave radiation emitted by the atmosphere, Cal/m²/hr
Q_c  Heat flux resulting from convection heat transfer, Cal/m²/hr
Q_e  Heat flux resulting from long-wave radiation emitted by the pavement surface, Cal/m²/hr
APPENDIX A (Cont)

\( Q_g \)  
Heat flux into the pavement, Cal/m²/hr

\( Q_h \)  
Heat flux from transpiration, condensation, evaporation and sublimation, Cal/m²/hr

\( Q_i \)  
Heat flux resulting from incident short-wave radiation, Cal/m²/hr

\( Q_r \)  
Heat flux resulting from reflected short-wave radiation, Cal/m²/hr

\( R \)  
Gas constant for water vapor, erg/g/C

\( T \)  
Temperature, C

\( t \)  
Time, sec.

\( x \) and \( y \)  
Horizontal coordinates, cm

\( z \)  
Vertical coordinate, cm

\( Z \)  
Water gravity term, cm

\( \alpha \)  
Tortuosity factor for diffusion of gases in soils

\( \beta \)  
g/cm³/C

\( \gamma \)  
Temperature coefficient, C⁻¹

\( \theta \)  
Volumetric water content, cm³/cm³

\( \lambda \)  
Thermal conductivity, Cal/cm/sec

\( \nu \)  
Mass flow factor

\( \rho \)  
Density of water vapor, gm/cm³

\( \rho_u \)  
Density of liquid water, g/cm³

\( \nabla \)  
Differential operator
APPENDIX B
Input to Moisture Model

A. Weather Data

B. Radiation Data

C. Introductory Variables
   1. Title
   2. Pavement Identification
   3. Weather Station
   4. Starting Year, Day, Time, and Temperature
   5. Finishing Year, Day, Time, and Temperature
   6. Iteration Time Increment
   7. Time Water Content and Temperatures are to be Printed
   8. End of Introductory Variables

D. Geometric Variables
   1. Pavement Surface Width
   2. Pavement Shoulder Width
   3. Distance to Ditch
   4. Water Table Depth
   5. Rate of Water Table Movement
   6. Ditch Front Slope
   7. Subsurface Drains
   8. Standing Water on Pavement Surface
   9. Standing Water in Ditch
   10. Centerline Gradient
   11. End of Geometric Variables

E. Model Variables
   1. One- or Two-Dimensional Program
   2. Vertical or Horizontal Moisture Movement if One-Dimensional
   3. Depth into Pavement to be Modeled
   4. Node Width
   5. Node Depth
   6. Number of Layers in the Pavement System
   7. End of Model Variables

F. Pavement Layer Properties
   1. Layer Number, Material Identification, Thickness
      Dry Density, and Specific Gravity for each
      Layer in Pavement System
   2. End of Layer Properties

G. Diffusivity vs Water Content Data
   1. Layer Number and Method of Inputting Data
      (Standard, Input, Fit, or Calculate)
   2. End of Diffusivity vs Water Content Data
H. Moisture Tension vs Water Content Data
   1. Layer, Number and Method of Inputting Data
      (Standard, Input, or Fit)
   2. End of Tension vs Water Content Data

I. Saturated Permeability
   1. Layer Number and Permeability
   2. End of Permeability Data

J. Surface and Precipitation Properties
   1. Rate of Rainfall
   2. Time of Rainfall
   3. Rate of Snowfall
   4. Time of Snowfall
   5. Equivalent Snow for Rain
   6. Percentage of Rain Runoff
   7. Pavement Surface Permeability
   8. End of Surface and Precipitation Properties

K. Thermal Properties
   1. Freezing Temperature
   2. Solar Emissivity and Absorbtivity at Pavement Surface
   3. Deep Soil Constant Temperature
   4. Depth to Constant Soil Temperature
   5. Upper and Lower Temperature of the Soil Freezing Range
   6. Radiation and Cloud Base Factors
   7. Depressed Freezing Point
   8. Thermal Conductivity for each Layer for Unfrozen, Freezing, and Frozen Conditions
   9. Heat Capacity for each Layer for Unfrozen, Freezing, and Frozen Conditions
   10. End of Thermal Properties

L. Initial Water Contents

M. Initial Temperature Distribution

N. Restrictions
   1. Isothermal
   2. Nonisothermal
   3. Isohumidity

O. Solve Model

* Data Considered in Future Moisture Model Expansion

Note: The input data used in a specified model run will depend on the model variables and restrictions.